GAS DYNAMICS AND JET PROPULSION

B.TECH. DEGREE COURSE

SCHEME AND SYLLABUS

(2002-03 ADMISSION ONWARDS)

MAHATMA GANDHI UNIVERSITY

KOTTAYAM,KERALA

GAS DYNAMICS AND JET PROPULSION

M 701

2+1+0

Module 1

Introduction to gas dynamics: control volume and system approaches acoustic waves and sonic velocity - Mach number - classification of fluid flow based on mach number - mach cone-compressibility factor - General features of one dimensional flow of a compressible fluid - continuity and momentum equations for a control volume.

Module 2

Isentropic flow of an ideal gas: basic equation - stagnation enthalpy, temperature, pressure and density-stagnation, acoustic speed - critical speed of sound- dimensionless velocity-governing equations for isentropic flow of a perfect gas - critical flow area - stream thrust and impulse function. Steady one dimensional isentropic flow with area change-effect of area change on flow parameters- chocking- convergent nozzle - performance of a nozzle under decreasing back pressure -De lavel nozzle - optimum area ratio effect of back pressure - nozzle discharge coefficients - nozzle efficiencies.

Module 3

Simple frictional flow: adiabatic flow with friction in a constant area duct-governing equations - fanno line limiting conditions - effect of wall friction on flow properties in an Isothermal flow with friction in a constant area duct-governing equations - limiting conditions. Steady one dimensional flow with heat transfer in constant area ducts- governing equations - Rayleigh line entropy change caused by heat transfer - conditions of maximum enthalpy and entropy

Module 4

Effect of heat transfer on flow parameters: Intersection of Fanno and Rayleigh lines. Shock waves in perfect gas- properties of flow across a normal shock - governing equations - Rankine Hugoniat equations - Prandtl's velocity relationship - converging diverging nozzle flow with shock thickness - shock strength.

Module 5

Propulsion: Air craft propulsion: - types of jet engines - energy flow through jet engines, thrust, thrust power and propulsive efficiency turbojet components-diffuser, compressor, combustion chamber, turbines, exhaust systems. Performance of turbo propeller engines, ramjet and pulsejet, scramjet engines. Rocket propulsion - rocket engines, Basic theory of equations - thrust equation - effective jet velocity - specific impulse - rocket engine performance - solid and liquid propellant rockets - comparison of various propulsion systems.

References

1. Compressible fluid flow - A. H. Shapiro
2. Fundamentals of compressible flow with aircraft and rocket propulsion - S. M. Yahya
3. Elements of gas dynamics - Liepman & Roshko
4. Aircraft & Missile propulsion - Zucrow
5. Gas dynamics - M.J. Zucrow & Joe D.Holfman

MODULE-1

CONCEPT OF GAS DYNAMICS

1.1. Introduction

Gas dynamics mainly concerned with the motion of gases and its effects .It differ from fluid dynamics .Gas dynamics considers thermal or chemical effects while fluid dynamics usually does not.

Gas dynamics deals with the study of compressible flow when it is in motion. It analyses the high speed flows of gases and vapors' with considering its compressibility. The term gas dynamics is very general and alternative names have been suggested e.g.: Supersonic flow, compressible flow and aero thermodynamics etc.,

1.2. Applications

Gas dynamics is of interest to both mechanical and the aeronautical engineers but particular field of interest of the two different .It may be said that thermodynamicist is concerned with how an object in motion influenced as it flies through still air. In contrast to it the thermodynamicist in more interested in the cases in which the object in stationary and the fluid is in motion .The applications of gas dynamics are given below.

1. It is used in Steam and Gas turbines
2. High speed aero dynamics
3. Jet and Rocket propulsion
4. High speed turbo compressor

The fluid dynamics of compressible flow problems which involves the relation between forse, density, velocity and mass etc.Therfore the following laws are frequently used for solving the dynamic problems.

1. Steady flow energy equation

2. Entropy relations

3. Continity equation

4. Momentum equation

1.3. One dimensional flow of a compressible fluid

It is meant that flow parameters changing in one direction only, particularly in direction of flow. In gas dynamics analysis it is necessary to satisfy four equations. These are conservation of mass, energy, Momentum along with the equations of the equation of the state of fluid used. In most practical applications the flow of a fluid through a pipe or duct can be approximated to be one dimensional flow and thus the properties can be assumed to vary in one direction only (the direction of flow).As a result , all the properties are assumed to have bulk average values over the cross section. However the values of the properties at a cross section may change with time unless the flow is steady.

The one dimensional-flow approximation has little impact on most properties of a fluid flowing in a pipe or duct such as temperature, pressure and density since these properties usually remain over the cross section .this is not the case of velocity, However, whose values varies from zero at the wall to maximum at the center because of the viscous effect (friction between fluid layers)

1.4. The Kinetic Molecular Theory

The kinetic molecular theory describes the properties of molecules in terms of motion (kinetic energy) and of temperature. The theory is most often applied to gases but is helpful in explaining molecular behavior in all states of matter. As applied to gases, the kinetic molecular theory has the following postulates:

1. Gases are composed of very tiny particles (molecules). The actual volume of these molecules is so small as to be negligible compared with the total volume of the gas sample. A gas sample is, then, mostly empty space. This fact explains the compressibility of gases.
2. There are no attractive forces between the molecules of a gas. This postulate explains why, over a period of time, the molecules of a gas do not cluster together at the bottom of its container.
3. The molecules of a gas are in constant, rapid, random, straight-line motion. This postulate explains why a gas spreads so rapidly through the available space - for

example, why the smell of hot coffee can spread quickly from the kitchen throughout the house.

4. During their motion, the gas molecules constantly collide with one another and with the walls of the container. (The collision with the walls provides the pressure exerted by a gas.) None of these collisions is accompanied by any loss of energy; instead, they are what is known as elastic collisions. A "new" tennis ball collides more elastically than a "dead" tennis ball.
5. The average kinetic energy of the molecules in a gas sample is proportional to its temperature (Kelvin) and is independent of the composition of the gas. In other words, at the same temperature, all gases have the same average kinetic energy. It also follows from this postulate that at zero Kelvin all molecular motion has ceased.

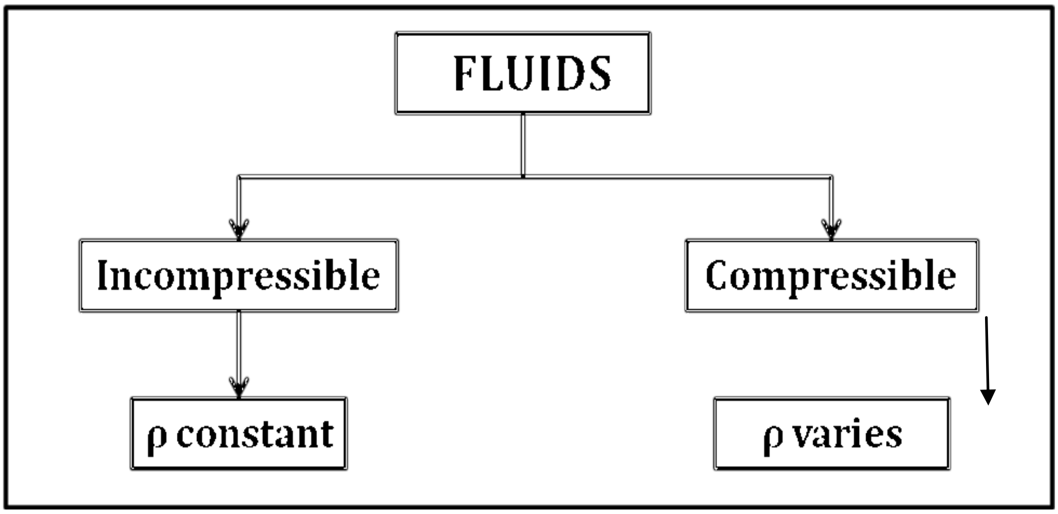
These postulates and the experimental evidence for them are summarized in Table

The kinetic molecular theory	
Postulate	**Evidence**
1. Gases are tiny molecules in mostly empty space.	The compressibility of gases.
2. There are no attractive forces between molecules.	Gases do not clump.
3. The molecules move in constant, rapid, random, straight-line motion.	Gases mix rapidly.
4. The molecules collide elastically with container walls and one another.	Gases exert pressure that does not diminish over time.
5. The average kinetic energy of the molecules is proportional to the Kelvin temperature of the sample.	Charles' Law

Clearly, the actual properties of individual gases vary somewhat from these postulates, for their molecules do have a real volume and there is some attraction between the molecules. However, our discussion will ignore these variations and concentrate on an ideal gas, one that behaves according to this model.

1.5. Introduction to compressible flows

- Compressible flow - Density changes



We know that fluids, such as gas, are classified as Incompressible and Compressible fluids. Incompressible fluids do not undergo significant changes in density as they flow. In general, liquids are incompressible; water being an excellent example. In contrast compressible fluids do undergo density changes. Gases are generally compressible; air being the most common compressible fluid we can find. Compressibility of gases leads to many interesting features such as shocks, which are absent for incompressible fluids. Gas dynamics is the discipline that studies the flow of compressible fluids and forms an important branch of Fluid Mechanics.

1.6.Compressible vs. Incompressible Flow

- A flow is classified as incompressible if the density remains nearly constant.
- Liquid flows are typically incompressible.
- Gas flows are often compressible, especially for high speeds.
- Mach number, Ma = V/c is a good indicator of whether or not compressibility effects are important.
  - Ma < 0.3 : Incompressible

– Ma < 1 : Subsonic

– Ma = 1 : Sonic

– Ma > 1 : Supersonic

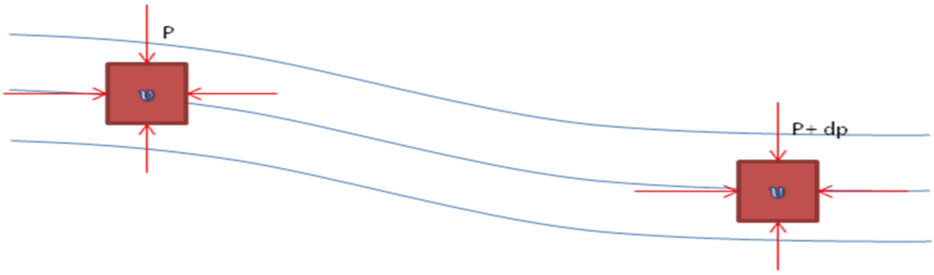
– Ma >> 1 : Hypersonic

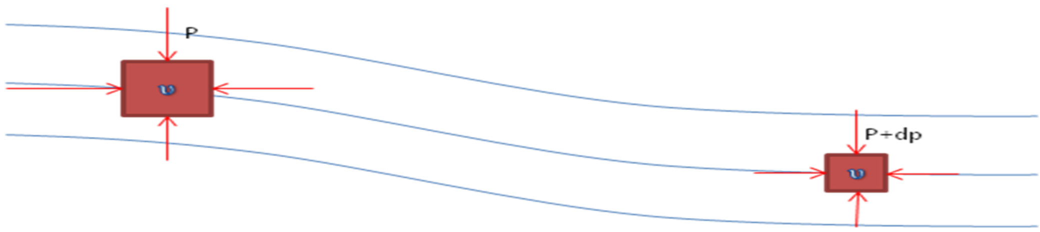
- Compressibility

Measure of the relative volume change with pressure

$$\tau = -\frac{1}{v}\frac{dv}{dP} = \frac{1}{\rho}\frac{d\rho}{dP}$$

A measure of the relative volume change with pressure for a given process. Consider a small element of fluid of volume v, the pressure exerted on the sides of the element is p. Assume the pressure is now increased by an infinitesimal amount dp. The volume of the element will change by a corresponding amount dv , here the volume decrease so dv is a negative quantity. By definition, the compressibility of fluid is





The terms compressibility and incompressibility describe the ability of molecules in a fluid to be compacted or compressed (made more dense) and their ability to bounce back to their original density, in other words, their "springiness." An incompressible fluid cannot be compressed and has relatively constant density throughout. Liquid is an incompressible fluid. A gaseous fluid such as air, on the other hand, can be either compressible or incompressible. Generally, for theoretical and experimental purposes, gases are assumed to be incompressible when they are moving at low speeds--under approximately 220 miles per hour. The motion of the object traveling through the air at such speed does not affect the density of the air. This assumption has been useful in aerodynamics when studying the behavior of air in relation to airfoils and other objects moving through the air at slower speeds.

In thermodynamics and fluid mechanics, compressibility is a measure of the relative volume change of a fluid or solid as a response to a pressure (or mean stress) change.

$$\beta = -\frac{1}{V}\frac{\partial V}{\partial p}$$

where V is volume and p is pressure. The above statement is incomplete, because for any object or system the magnitude of the compressibility depends strongly on whether the process is adiabatic or isothermal. Accordingly we define the isothermal compressibility as:

$$\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

Where the subscript T indicates that the partial differential is to be taken at constant temperature. The adiabatic compressibility as:

$$\beta_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$$

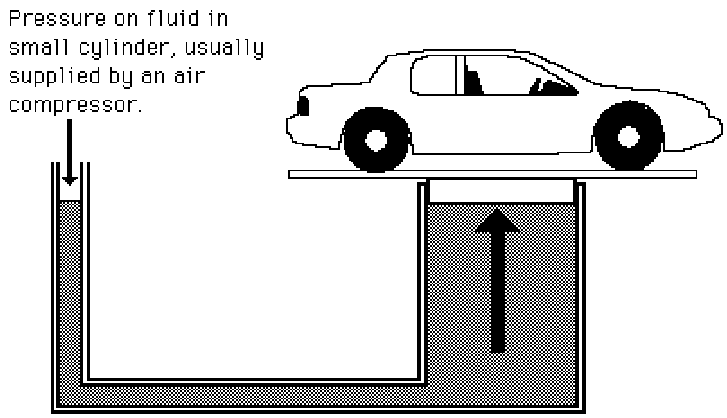
where S is entropy. For a solid, the distinction between the two is usually negligible.

The inverse of the compressibility is called the bulk modulus, often denoted K (sometimes B).

1.7. Compressibility and Incompressibility

The terms compressibility and incompressibility describe the ability of molecules in a fluid to be compacted or compressed (made more dense) and their ability to bounce back to their original density, in other words, their "springiness." An incompressible fluid cannot be compressed and has relatively constant density throughout. Liquid is an incompressible fluid. A gaseous fluid such as air, on the other hand, can be either compressible or incompressible. Generally, for theoretical and experimental purposes, gases are assumed to be incompressible when they are moving at low speeds--under approximately 220 miles per hour. The motion of the object traveling through the air at such speed does not affect the density of the air. This assumption has been useful in aerodynamics when studying the behavior of air in relation to airfoils and other objects moving through the air at slower speeds.

However, when aircraft began traveling faster than 220 miles per hour, assumptions regarding the air through which they flew that were true at slower speeds were no longer valid. At high speeds some of the energy of the quickly moving aircraft goes into compressing the fluid (the air) and changing its density. The air at higher altitudes where these aircraft fly also has lower density than air nearer to the Earth's surface. The airflow is now compressible, and aerodynamic theories have had to reflect this. Aerodynamic theories relating to compressible airflow characteristics and behavior are considerably more complex than theories relating to incompressible airflow. The noted aerodynamicist of the early 20th century, Ludwig Prandtl, contributed the Prandtl-Glaubert rule for subsonic airflow to describe the compressibility effects of air at high speeds. At lower altitudes, air has a higher density and is considered incompressible for theoretical and experimental purposes.



Compressibility

- Compressibility of any substance is the measure of its change in volume under the action of external forces.

- The normal compressive stress on any fluid element at rest is known as hydrostatic pressure p and arises as a result of innumerable molecular collisions in the entire fluid.

- The degree of compressibility of a substance is characterized by the **bulk modulus of elasticity** E defined as

$$E = \lim_{\Delta\forall \to 0} \left(\frac{-\Delta p}{\Delta\forall \,/\, \forall} \right)$$

- Where $\Delta\forall$ and Δp are the changes in the volume and pressure respectively, and $\forall$is the initial volume. The negative sign (-sign) is included to make E positive, since increase in pressure would decrease the volume i.e for $\Delta p>0$, $\Delta\forall<0$) in volume.
- For a given mass of a substance, the change in its volume and density satisfies the relation

$$\Delta m = 0, \ \Delta \ (\rho\forall) = 0$$

$$\frac{\Delta\forall}{\forall} = -\frac{\Delta\rho}{\rho}$$

using

$$E = \lim_{\Delta\forall \to 0} \left(\frac{-\Delta p}{\Delta\forall \,/\, \forall} \right) \ \ \& \ \frac{\Delta\forall}{\forall} = -\frac{\Delta\rho}{\rho}$$

we get

$$E = \lim_{\Delta\rho \to 0}\left(\frac{\Delta p}{\Delta\rho/\rho}\right) = \rho\frac{dp}{d\rho}$$

- Values of *E* for liquids are very high as compared with those of gases (except at very high pressures). Therefore, liquids are usually termed as incompressible fluids though, in fact, no substance is theoretically incompressible with a value of *E* as ∞ .

- For example, the bulk modulus of elasticity for water and air at atmospheric pressure are approximately 2 x 10^6 kN/m ^{2} and 101 kN/m ^{2} respectively. It indicates that air is about 20,000 times more compressible than water. Hence water can be treated as incompressible.

- For gases another characteristic parameter, known as compressibility *K*, is usually defined , it is the reciprocal of *E*

$$K = \frac{1}{E} = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right) = -\frac{1}{\forall}\left(\frac{d\forall}{dp}\right)$$

- K is often expressed in terms of specific volume $\forall$.
- For any gaseous substance, a change in pressure is generally associated with a change in volume and a change in temperature simultaneously. **A** functional relationship between the pressure, volume and temperature at any equilibrium state is known as thermodynamic equation of state for the gas.
  For an ideal gas, the thermodynamic equation of state is given by

$$p = \rho RT$$

- where *T* is the temperature in absolute thermodynamic or gas temperature scale (which are, in fact, identical), and *R* is known as the characteristic gas constant, the value of which depends upon a particular gas. However, this equation is also valid for the real gases which are thermodynamically far from their liquid phase. For air, the value of R is 287 J/kg K.

Nature of Process

- The relationship between the pressure *p* and the volume V for any process undergone by a gas depends upon the nature of the process. A general relationship is usually expressed in the form of

$$p\forall^{x} = constant \qquad (2.8)$$

This is an equation of state of a polytropic process. For a constant temperature (isothermal) process of an ideal gas, x= 1.

- If there is no heat transfer to or from the gas, the process is known as **adiabatic**. A frictionless adiabatic process is called an **isentropic** process and x equals to the ratio of specific heat at constant pressure to that at constant volume.

The equation PV^x=constant can be written in a differential form as

$$\frac{d\forall}{dp} = -\frac{\forall}{xp}$$

Using the relations

$$\frac{d\forall}{dp} = -\frac{\forall}{xp}$$

$$E = \lim_{\Delta\rho \Rightarrow 0} \frac{\Delta p}{\Delta\rho / \rho} = \rho \frac{dp}{d\rho}$$

and

$$K = 1 / E = \frac{1}{\rho}\frac{d\rho}{dp} = \frac{1}{\forall}\left[\frac{d\forall}{dp}\right]$$

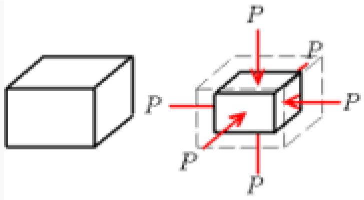
We get

$$E = xp \quad or \quad K = \frac{1}{xp}$$

Therefore, the compressibility ***K,*** or bulk modulus of elasticity ***E*** for gases depends on the nature of the process through which the pressure and volume change**.**

1.8. Bulk modulus (K)

The **bulk modulus** (K) of a substance measures the substance's resistance to uniform compression. It is defined as the pressure increase needed to cause a given relative decrease in volume. Its base unit is Pascal



As an example, suppose an iron cannon ball with bulk modulus 160 GPa is to be reduced in volume by 0.5%. This requires a pressure increase of
0.005×160 GPa = 0.8 GPa (116,000 psi).

The bulk modulus K can be formally defined by the equation:

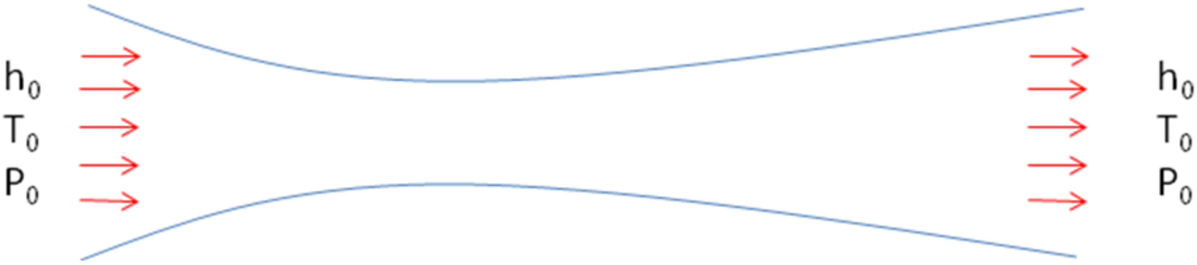
$$K = -V\frac{\partial P}{\partial V}$$

where P is pressure, V is volume, and $\partial P/\partial V$ denotes the partial derivative of pressure with respect to volume. The inverse of the bulk modulus gives a substance's compressibility.

Other moduli describe the material's response (strain) to other kinds of stress: the shear modulus describes the response to shear, andYoung's modulus describes the response to linear strain. For a fluid, only the bulk modulus is meaningful. For an anisotropic solid such as wood or paper, these three moduli do not contain enough information to describe its behaviour, and one must use the full generalized Hooke's.

1.8. Isentropic Flow

- Adiabatic and Reversible
- No energy added, No energy losses
- Small an gradual change in flow variables



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An **isentropic flow** is a flow that is both adiabatic and reversible, that is no energy is added to the flow, and no energy losses occur due to friction or dissipative effects

Isentropic flows occur when the change in flow variables is small and gradual, such as the ideal flow through the nozzle shown below. Considering flow through a tube, as shown in the figure, if the flow is very gradually compressed (area decreases) and then gradually expanded (area increases), the flow conditions return to their original values. We say that such a process is **reversible**. From a consideration of the second law of thermodynamics, a reversible flow maintains a constant value of entropy. Engineers call this type of flow an **isentropic** flow.

As a gas is forced through a tube, the gas molecules are deflected by the walls of the tube. If the speed of the gas is much less than the speed of sound of the gas, the density of the gas remains constant and the velocity of the flow increases. However, as the speed of the flow approaches the speed of sound we must consider compressibility effects on the gas. The density of the gas varies from one location to the next.

The generation of sound waves is an isentropic process. A supersonic flow that is turned while the flow area increases is also isentropic. We call this an isentropic expansion because of the area increase. If a supersonic flow is turned abruptly and the flow area decreases, shock waves are generated and the flow is **irreversible**. The isentropic relations are no longer valid and the flow is governed by the oblique or normal shock relations.

On this slide we have collected many of the important equations which describe an isentropic flow. We begin with the definition of the Mach number since this parameter appears in many of the isentropic flow equations. The Mach number **M** is the ratio of the speed of the flow **v** to the speed of sound **a**.

1.9. Concept of Continuum

The concept of continuum is a kind of idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space variables. Although any matter is composed of several molecules, the concept of continuum assumes a continuous distribution of mass within the matter or system with no empty space, instead of the actual conglomeration of separate molecules.

Describing a fluid flow quantitatively makes it necessary to assume that flow variables (pressure , velocity etc.) and fluid properties vary continuously from one point to another. Mathematical description of flow on this basis have proved to be reliable and treatment of fluid medium as a continuum has firmly become established. For example density at a point is normally defined as

$$\rho = \lim_{\Delta\forall \to 0} \left(\frac{m}{\Delta\forall} \right)$$

Here $\Delta\forall$ is the volume of the fluid element and m is the mass

If $\Delta\forall$ is very large ρ is affected by the in homogeneities in the fluid medium. Considering another extreme if $\Delta\forall$ is very small, random movement of atoms (or molecules) would change their number at different times. In the continuum approximation point density is defined at the smallest magnitude of $\Delta\forall$, before statistical fluctuations become significant. This is called continuum limit and is denoted by $\Delta\forall_c$.

$$\rho = \lim_{\Delta\forall \to \Delta\forall_C} \left(\frac{m}{\Delta\forall} \right)$$

One of the factors considered important in determining the validity of continuum model is molecular density. It is the distance between the molecules which is characterised by mean free path (λ). It is calculated by finding statistical average distance the molecules travel between two successive collisions. If the mean free path is very small as compared with some characteristic length in the flow domain (i.e., the molecular density is very high) then the gas can be treated as a continuous medium. If the mean free path is large in comparison to some characteristic length, the gas cannot be considered continuous and it should be analysed by the molecular theory.

A dimensionless parameter known as **Knudsen number**, $K_n = \lambda / L$, where λ is the mean free path *and L* is the characteristic length. It describes the degree of departure from continuum.

Usually when $K_n > 0.01$, the concept of continuum does not hold good.

Beyond this critical range of Knudsen number, the flows are known as

slip flow ($0.01 < K_n < 0.1$),

transition flow ($0.1 < K_n < 10$) and

free-molecule flow ($K_n > 10$).

However, for the flow regimes considered in this course , K_n is always less than 0.01 and it is usual to say that the fluid is a continuum. Other factor which checks the validity of continuum is the elapsed time between collisions. The time should be small enough so that the random statistical description of molecular activity holds good. In continuum approach, fluid properties such as density, viscosity, thermal conductivity, temperature, etc. can be expressed as continuous functions of space and time.

Matter may be described at a molecular (or microscopic) level using the techniques of statistical mechanics and kinetic theory. For engineering purposes, however, we want ``averaged" information, i.e., a macroscopic, not a microscopic, description. There are two reasons for this. First, a microscopic description of an engineering device may produce too much information to manage. For example, 1 mm^3 of air at standard temperature and pressure contains 10^{16} molecules , each of which has a position and a velocity. Typical engineering applications involve more than 10^{20} molecules. Second, and more importantly, microscopic positions and velocities are generally not useful for determining how macroscopic systems will act or react unless, for instance, their total effect is integrated. We therefore neglect the fact that real substances are composed of discrete molecules and model matter from the start as a smoothed-out **continuum**. The information we have about a continuum represents the microscopic information averaged over a volume. **Classical thermodynamics** is concerned only with continua.

1.10. Equations of state

It is an experimental fact that two properties are needed to define the state of any pure substance in equilibrium or undergoing a steady or quasi-steady process. Thus for a simple compressible gas like air,

$$P = P(v, T), \quad \text{or} \quad v = v(P, T), \quad \text{or} \quad T = T(P, v),$$

Where v is the volume per unit mass,

$$1/\rho$$

.

In words, if we know v and T we know P, etc.

Any of these is equivalent to an equation

$$f(P, v, T) = 0$$

,

Which is known as an equation of state? The equation of state for an ideal gas, which is a very good approximation to real gases at conditions that are typically of interest for aerospace applications[1.2], is

$$P\bar{v} = \mathcal{R}T,$$

where $\bar{v}$ is the volume per mol of gas and

$\mathcal{R}$ is the ``Universal Gas Constant,''

$$8.31 \text{ kJ/kmol-K}$$

.

A form of this equation which is more useful in fluid flow problems is obtained if we divide by the molecular weight, $\mathcal{M}$:

$$Pv = RT, \quad \text{or} \quad P = \rho RT$$

where R is

$$\mathcal{R}/\mathcal{M}$$

,

which has a different value for different gases due to the different molecular weights. For air at room conditions,

$$R = 0.287 \text{ kJ/kg-K}$$

.

Enthalpy

In many thermodynamic fluid process analyses the sum of the internal energy (U) and the product of pressure (P) and volume (V) is present. The combination (U + PV) is called the enthalpy of the fluid. H is a thermodynamic fluid property but is does not have an absolute value(because it includes internal energy U)value and therefore enthalpy changes are generally applied or enthalpy values are identified relative to a fixed state e.g. water at 273 deg.K . It is important to note that enthalpy is simply a combination of properties..it is not a form of stored energy although for certain applications it can be treated as energy.

$$H = U + PV \text{ ....(extensive property)}$$

per unit mass

$$h = u + Pv \text{ ....(intensive property)}$$

Gas Constant R

The gas constant R is derived from the equation of state

$$Pv = RT \text{ .. for unit mass of gas}$$

$$PV = mRT$$

The gas constant R is different for each gas and has different units depending on the unit systems used. Typical units are (kJ/kg.K).

The universal gas constant R_u is the same for all gases and is defined by

$$PV = NR_uT$$

- R = Gas Constant = R_u /M
- R_u = Universal Gas Constant
- v = Gas volume (m^3)
- V = Gas Volume (m^3)
- N = Number of Moles
- T = Absolute Temperature deg K
- M = Molar mass (kg)
- P = Absolute Pressure N/m^3 (kg)

Internal Energy and Enthalpy

Microscopic view of a gas is a collection of particles in random motion. Energy of a particle consists of **translational energy, rotational energy, vibrational energy** and **specific electronic energy**. All these energies summed over all the particles of the gas, form the specific internal energy, e , of the gas.

- Imagine a gas in thermodynamic equilibrium,i.e., gradients in velocity, pressure, temperature and chemical concentrations do not exist.

Then the enthalpy, h , is defined as

$$h = e + p\upsilon \,,$$

Where *v* is the specific volume.

$$e = e(T, \forall)$$

$$h = h(T, p)$$

If the gas is not chemically reacting and the intermolecular forces are neglected, the system can be called as a **thermally perfect gas**, where internal energy and enthalpy are functions of temperature only. One can write

$$e = e(T)$$

$$h = h(T)$$

$$de = c_{\nu} dT$$

$$dh = c_p dT$$

For a calorically perfect gas,

$$e = c_{\nu} T$$

$$e = c_{\nu} T$$

$$h = c_p T$$

$$e = c_{\nu} T$$

$$h = c_p T$$

Please note that in most of the compressible flow applications, the pressure and temperatures are such that the gas can be considered as calorically perfect.

- For calorically perfect gases, we assume constant specific heats and write

$$c_p - c_v = R$$

- The specific heats at constant pressure and constant volume are defined as

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p \qquad c_v = \left(\frac{\partial e}{\partial T}\right)_v$$

Equation, can be rewritten as

$$1 - \frac{c_v}{c_p} = \frac{R}{c_p}$$

Also

$$\frac{c_p}{c_v} = \gamma$$

.

So we can rewrite Eq. as

$$1 - \frac{1}{\gamma} = \frac{R}{c_p}$$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

In a similar way, from Eq. we can write

$$c_v = \frac{R}{\gamma - 1}$$

First Law of Thermodynamics...

This law expresses the general law of conservation of energy. and states that heat and work are mutually convertible

Heat In = Work Out over complete cycle
or Sum (d Q) = sum (d W)

Over a complete cycle the algebraic sum of the quantities of heat supplied to a system is equal to the algebraic sum quantities of work performed by the system i.e.

$$\oint (\delta Q - \delta W) = 0$$

In a cyclic process any property of the system are the same at the end of a cycle as at the beginning. Throughout the path of a cycle ($\delta Q - \delta W$) represents a change in the total stored internal energy property of the system δE . The basic energy equation results from this

$$\delta Q = \delta E + \delta W$$

The total stored internal energy E includes for various forms of energy including

- the various forms of potential energy (gravity, magnetic, electrical)
- thermal energy
- chemical energy
- atomic energy
- kinetic energy
- surface tension energy

Note: In classical thermodynamics as applicable to mechanical engineering the atomic energy and the chemical energy are not considered....

$$E = U + P.E + K.E + S.E$$

P.E = total potential energy, K.E = total kinetic energy, S.E = total surface energy . The intrinsic internal energy U is the total internal energy minus the energies of motion, gravitational, magnetic and surface forces energies . The first law can be written using U as

$$\delta Q = \delta U + \delta W$$

This is termed the restricted energy conservation equation for a system. U is dependent on temperature and is not dependent on pressure or volume.

Control volume form of the conservation laws

The thermodynamic laws (as well as Newton's laws) are for a system, a specific quantity of matter. More often, in propulsion and power problems, we are interested in what happens in a fixed volume, for example a rocket motor or a jet engine through which mass is flowing at a certain *rate*. We may also be interested in the *rates* of heat and work into and out of a system. For this reason, the control volume form of the system laws is of great importance. A schematic of the difference is shown in Figure 1. Rather than focus on a particle of mass which moves through the engine, it is more convenient to focus on the volume occupied by the engine. This requires us to use the control volume form of the thermodynamic laws, developed below.

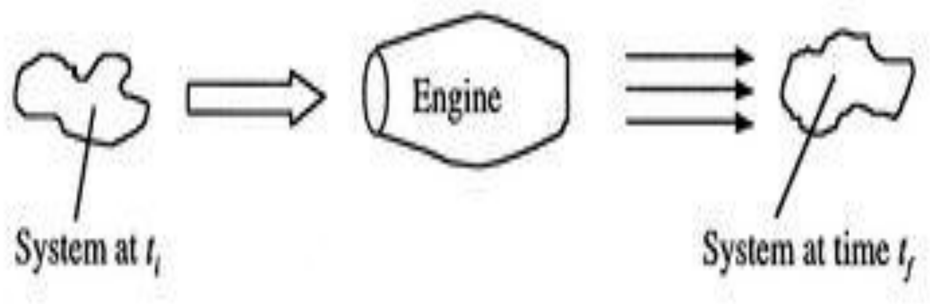


Fig 1.3. Control volume and system for flow through a propulsion device

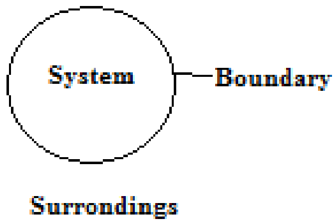
Q. **System :**

A thermodynamic system is defined as a quantity of matter or a region in space upon which attention is concentrated in the analysis of a problem. Everything external to the system is called the surroundings or the environment. The system is separated from the surroundings or the system boundary. The boundary may be either fixed or moving. A system and its surroundings together comprise a may be either fixed or moving.

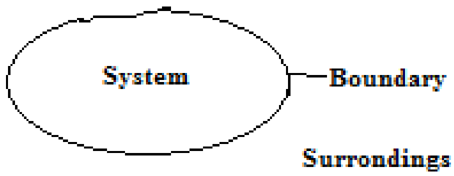
There are three types of system:

a) Closed system: A system of fixed mass. There is no transfer of mass across the system boundary. There may be energy transfer into or out of the system.
b) Open system: One in which matter and energy crosses the boundary of the system.
c) Isolated system: One in which there is no interaction between the system and the surroundings. It is of fixed mass and energy, and there is no mass or energy transfer.

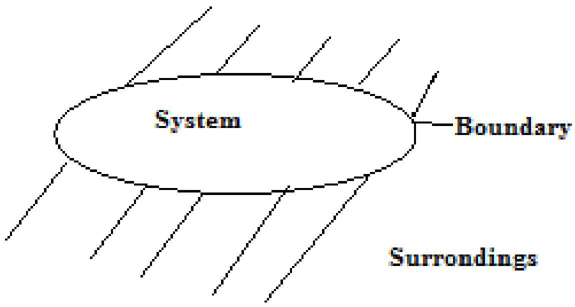
Open System



Closed System



Isolated System



Control volume:

A certain volume in space surrounding the system bounded by a surface called the control surface is known as the control volume.

Conservation of mass

For the control volume shown, the rate of change of mass inside the volume is given by the difference between the mass flow rate in and the mass flow rate out. For a single flow coming in and a single flow coming out this is

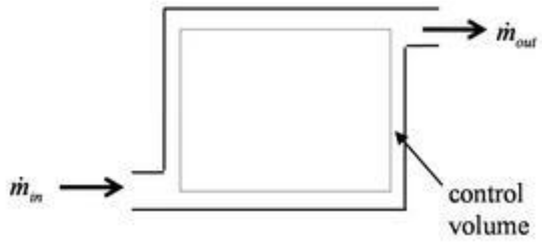
$$\frac{dm_{\text{cv}}}{dt} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}}$$

If the mass inside the control volume changes with time it is because some mass is added or some is taken out. In the special case of a steady flow,

$$d/dt = 0$$

Therefore ,

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m}.$$



A control volume used to track mass flows

Conservation of energy

The first law of thermodynamics can be written as a rate equation:

$$\frac{dE}{dt} = \dot{Q} - \dot{W},$$

where

$$\dot{Q} = \lim_{dt \to 0} \left(\frac{\delta Q}{dt} \right) \qquad \text{rate of total heat transfer to the system}$$

$$\dot{W} = \lim_{dt \to 0} \left(\frac{\delta W}{dt} \right) \qquad \text{rate of total work done by the system.}$$

To derive the first law as a rate equation for a *control volume* we proceed as with the mass conservation equation. The physical idea is that any rate of change of energy in the control volume must be caused by the rates of energy flow into or out of the volume. The heat transfer and the work are already included and the only other contribution must be associated with the mass flow in and out, which carries energy with it. Figure 2.10 shows two schematics of this idea. The desired form of the equation will be

$$(\text{rate of chang} = (\text{rate of heat added to C.V.}) - (\text{rate of work done}) +$$

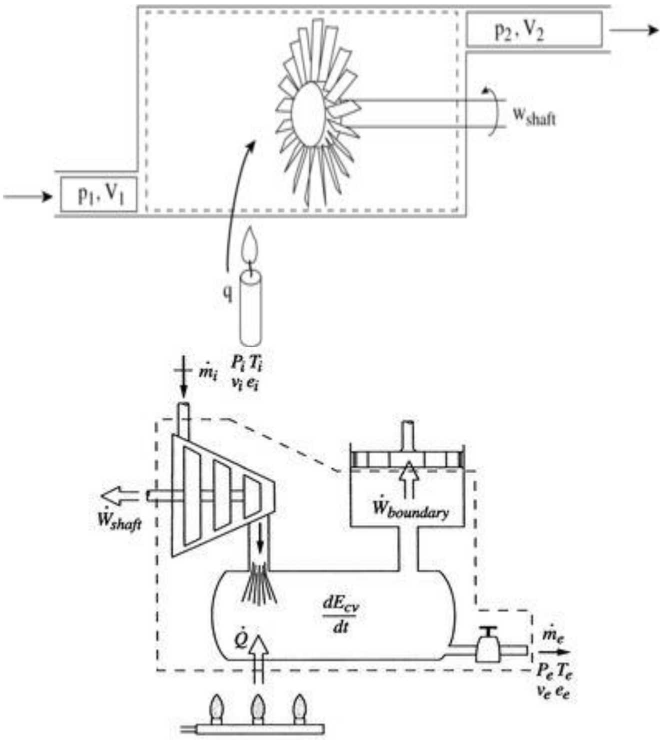


Figure 1.4: Schematic diagrams illustrating terms in the energy equation for a simple and a more general control volume

The fluid that enters or leaves has an amount of energy per unit mass given by

$$e = u + c^2/2 + gz,$$

Where c is the fluid velocity relative to some coordinate system, and we have neglected chemical energy. In addition, whenever fluid enters or leaves a control volume there is a work term associated with the entry or exit. We saw this in Section 2.3, example 1, and the present derivation is essentially an application of the ideas presented there. Flow exiting at station ``e" must push back the surrounding fluid, doing work on it. Flow entering the volume at station ``i" is pushed on by, and receives work from the surrounding air. The rate of flow work at exit is given by the product of the pressure times the exit area times the rate at which the external flow is ``pushed back." The latter, however, is equal to the volume per unit mass times the rate of mass flow. Put another way, in a time dt, the work done on the surroundings by the flow at the exit station is

$$dW_{\text{flow}} = pvdm_e.$$

The net rate of flow work is

$$\dot{W}_{\text{flow}} = p_e v_e \dot{m}_e - p_i v_i \dot{m}_i.$$

Including all possible energy flows (heat, shaft work, shear work, piston work, etc.), the first law can then be written as:

$$\frac{d}{dt}\sum E_{\mathrm{cv}} = \sum \dot{Q}_{\mathrm{cv}} + \sum \dot{W}_{\mathrm{shaft}} + \sum \dot{W}_{\mathrm{shear}} + \sum \dot{W}_{\mathrm{piston}} + \sum \dot{W}_{\mathrm{flow}} + \sum \dot{m}\left(u + \frac{c^2}{2} + gz\right)$$

where $\sum$ includes the sign associated with the energy flow. If heat is added or work is done *on* the system then the sign is positive, if work or heat are extracted *from* the system then the sign is negative. NOTE: this is consistent with

$$\Delta E = Q - W$$

,

where W is the work done *by* the system on the environment, thus work is flowing out of the system.We can then combine the specific internal energy term, u, in e and the specific flow work term, *pv* to make the enthalpy appear:

Total energy associated with mass flow:

$$e + pv = u + c^2/2 + gz + pv = h + c^2/2 + gz.$$

Thus, the first law can be written as:

$$\frac{d}{dt}\sum E_{\mathrm{cv}} = \sum \dot{Q}_{\mathrm{cv}} + \sum \dot{W}_{\mathrm{shaft}} + \sum \dot{W}_{\mathrm{shear}} + \sum \dot{W}_{\mathrm{piston}} + \sum \dot{m}\left(h + \frac{c^2}{2} + gz\right).$$

For most of the applications in this course, there will be no shear work and no piston work. Hence, the first law for a control volume will be most often used as:

$$\frac{dE_{\mathrm{cv}}}{dt} = \dot{Q}_{\mathrm{cv}} - \dot{W}_{\mathrm{shaft}} + \dot{m}_i\left(h_i + \frac{c_i^2}{2} + gz_i\right) - \dot{m}_e\left(h_e + \frac{c_e^2}{2} + gz_e\right).$$

Note how our use of enthalpy has simplified the rate of work term. In writing the control volume form of the equation we have assumed only one entering and one leaving stream, but this could be generalized to any number of inlet and exit streams.

In the special case of a steady-state flow,

$$\frac{d}{dt} = 0 \qquad \text{and} \qquad \dot{m}_i = \dot{m}_e = \dot{m}.$$

Applying this to Equation 2.10 produces a form of the ``Steady Flow Energy Equation'' (SFEE),

$$\dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} = \dot{m}\left[\left(h_e + \frac{c_e^2}{2} + gz_e\right) - \left(h_i + \frac{c_i^2}{2} + gz_i\right)\right],$$

which has units of Joules per second. We could also divide by the mass flow to produce

$$q_{\text{cv}} - w_{\text{cv}} = \left(h_e + \frac{c_e^2}{2} + gz_e\right) - \left(h_i + \frac{c_i^2}{2} + gz_i\right),$$

which has units of Joules per second per kilogram. For problems of interest in aerospace applications the velocities are high and the term that is associated with changes in the elevation is small. From now on, we will neglect the *gz* terms unless explicitly stated.

Combined First and Second Law Expressions

The first law, written in a form that is always true:

$$dU = dQ - dW.$$

For **reversible processes only**, work or heat may be rewritten as

$$dW = PdV,$$

$$dQ = TdS.$$

Substitution leads to other forms of the first law **true for reversible processes only**:

$$dU = dQ - \underline{PdV},\ \text{substituted for a reversible } dW$$

$$dU = \underline{TdS} - dW,\ \text{substituted for a reversible } dQ.$$

(If the substance has other work modes, e.g., stress, strain,

$$dU = dQ - PdV - XdY,$$

where X is a pressure-like quantity, and Y is a volume-like quantity.)

Substituting for both dW and dQ in terms of state variables,

$$dU = TdS - PdV \qquad \text{Always true.}$$

The above is always true because it is a relation between properties and is now independent of process. In terms of specific quantities:

$du = Tds - Pdv$ **Combined first and second law (a) or Gibbs equation (a).**

The combined first and second law expressions are often more usefully written in terms of the enthalpy, or specific enthalpy, $h = u + Pv$,

$$\begin{aligned} dh &= \underline{du} + Pdv + vdP \\ &= \underline{Tds - Pdv} + Pdv + vdP, \text{ using the first law.} \\ dh &= Tds + vdP. \end{aligned}$$

Or, since $v = 1/\rho$,

$dh = Tds + \dfrac{dP}{\rho}$ **Combined first and second law (b) or Gibbs equation (b).**

In terms of enthalpy (rather than specific enthalpy) the relation is

$$dH = TdS + VdP.$$

Entropy Changes in an Ideal Gas

Many aerospace applications involve flow of gases (e.g., air) and we thus examine the entropy relations for ideal gas behavior. The starting point is form (a) of the combined first and second law,

$$du = Tds - Pdv.$$

For an ideal gas,

$$du = c_v dT$$

.

Thus

$$Tds = c_v dT + P dv \quad \text{or} \quad ds = c_v \frac{dT}{T} + \frac{P}{T} dv.$$

Using the equation of state for an ideal gas ($Pv = RT$), we can write the entropy change as an expression with only exact differentials:

$$ds = c_v \frac{dT}{T} + R\frac{dv}{v}.$$

We can think of Equation (5.2) as relating the fractional change in temperature to the fractional change of volume, with scale factors c_v and R; if the volume increases without a proportionate decrease in temperature (as in the case of an adiabatic free expansion), then s increases. Integrating Equation (5.2) between two states ``1" and ``2":

$$\Delta s = s_2 - s_1 = \int_{T_1}^{T_2} c_v \frac{dT}{T} + R \int_{v_1}^{v_2} \frac{dv}{v}.$$

For a perfect gas with constant specific heats

$$\Delta s = s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right).$$

In non-dimensional form (using $R/c_v = (\gamma - 1)$)

$$\frac{\Delta s}{c_v} = \ln\left(\frac{T_2}{T_1}\right) + (\gamma - 1)\ln\left(\frac{v_2}{v_1}\right), \qquad \text{Entropy change of a perfect gas.}$$

Equation 5.3 is in terms of specific quantities. For N moles of gas,

$$\frac{\Delta S}{C_v} = N\left[\ln\left(\frac{T_2}{T_1}\right) + (\gamma - 1)\ln\left(\frac{V_2}{V_1}\right)\right].$$

This expression gives entropy change in terms of temperature and volume. We can develop an alternative form in terms of pressure and volume, which allows us to examine an assumption we have used. The ideal gas equation of state can be written as

$$\ln P + \ln v = \ln R + \ln T.$$

Taking differentials of both sides yields

$$\frac{dP}{P} + \frac{dv}{v} = \frac{dT}{T}.$$

Using the above equation in Eq. , and making use of the relations

$$c_p = c_v + R$$

;

$$c_p/c_v = \gamma$$

,

we find

$$ds = c_v\left[\frac{dP}{P} + \frac{dv}{v}\right] + R\frac{dv}{v},$$

or

$$\frac{ds}{c_v} = \frac{dP}{P} + \gamma\frac{dv}{v}.$$

Integrating between two states 1 and 2

$$\frac{\Delta s}{c_v} = \ln\left(\frac{P_2}{P_1}\right) + \gamma\ln\left(\frac{v_2}{v_1}\right) = ln\left[\frac{P_2}{P_1}\left(\frac{v_2}{v_1}\right)^\gamma\right].$$

Using both sides of (5.4) as exponents we obtain

$$\frac{P_2 v_2^\gamma}{P_1 v_1^\gamma} = [Pv^\gamma]_1^2 = e^{\Delta s/c_v}.$$

Equation above describes a general process. For the specific situation in which $\Delta s = 0$, i.e., the entropy is constant, we recover the expression $Pv^{\gamma} = \text{constant}$. It was stated that this expression applied to a reversible, adiabatic process. We now see, through use of the second law, a deeper meaning to the expression, and to the concept of a reversible adiabatic process, in that both are characteristics of a constant entropy, or **isentropic**, process.

Reversible adiabatic processes for an ideal gas

From the first law, with

$Q = 0$, $du = c_v dT$, and $\text{Work} = pdv$,

$$du + pdv = 0.$$

Also, using the definition of enthalpy,

$$dh = \underline{du + pdv} + vdp.$$

The underlined terms are zero for an adiabatic process. Rewriting (2.7) and (2.8),

$$\gamma c_v dT = -\gamma pdv$$

$$c_p dT = vdp.$$

Combining the above two equations we obtain

$$-\gamma pdv = vdp \quad \text{or} \quad -\gamma dv/v = dp/p.$$

Equation (2.9) can be integrated between states 1 and 2 to give

$$-\gamma ln(v_2/v_1) = ln(p_2/p_1), \text{ or, equivalently,}$$

$$(p_2 v_2^{\gamma})/(p_1 v_1^{\gamma}) = 1.$$

For an *ideal gas* undergoing a *reversible, adiabatic process*, the relation between pressure and volume is thus:

$$pv^{\gamma} = \text{constant, or}$$

$$p = \text{constant} \times \rho^{\gamma}.$$

We can substitute for p or v in the above result using the ideal gas law, or carry out the derivation slightly differently, to also show that

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{and} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

We will use the above equations to relate pressure and temperature to one another for quasi-static adiabatic processes (for instance, this type of process is our idealization of what happens in compressors and turbines).

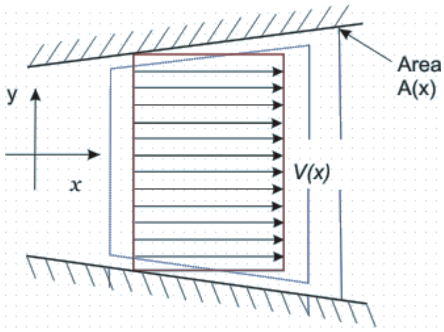
Basic Equations for One-Dimensional Flow

- Here we will study a class of compressible flows that can be treated as one dimensional flow. Such a simplification is meaningful for flow through ducts where the centreline of the ducts does not have a large curvature and the cross-section of the ducts does not vary abruptly.

- In one dimension, the flow can be studied by ignoring the variation of velocity and other properties across the normal direction of the flow. However, these distributions are taken care of by assigning an average value over the cross-section (Fig. 39.3).

- The area of the duct is taken as A(x) and the flow properties are taken as p(x), ρ(x), V(x) etc. The forms of the basic equations in a one-dimensional compressible flow are;

    - **Continuity Equation**
    - **Energy Equation**
    - **Bernoulli and Euler Equations**
    - **Momentum Principle for a Control Volume**



Continuity Equation

For steady one-dimensional flow, the equation of continuity is

$$\rho(x).\mathrm{V}(x).\mathrm{A}(x) = const$$

Differentiating(after taking log), we get

$$\frac{d\rho}{\rho} + \frac{d\mathrm{V}}{\mathrm{V}} + \frac{d\mathrm{A}}{\mathrm{A}} = 0$$

Energy Equation

Consider a control volume within the duct shown by dotted lines in Fig. The first law of thermodynamics for a control volume fixed in space is

$$\frac{d}{dt}\iiint \rho(\mathrm{e} + \frac{\mathrm{V}^2}{2})d\forall + \iint (\mathrm{e} + \frac{\mathrm{V}^2}{2})\,\rho\mathrm{V}\,d\mathrm{A} = \iint \mathrm{V}(\tau.\,d\mathrm{A}) - \iint \mathrm{q}.\,d\mathrm{A}$$

Where

$$\frac{\mathrm{V}^2}{2}$$

is the kinetic energy per unit mass.

Let us discuss the various terms from above equation:

- The first term on the left hand side signifies the rate of change of energy (internal + kinetic) within the control volume
- The second term depicts the flux of energy out of control surface.
- The first term on the right hand side represents the work done on the control surface
- The second term on the right means the heat transferred through the control surface.

It is to be noted that dA is directed along the outward normal.

- Assuming steady state, the first term on the left hand side of Eq. (39.7) is zero. Writing $\rho_2 V_2 A_2 = \rho_1 V_1 A_1 = \dot{m}$ (where the subscripts are for Sections 1 and 2), the second term on the left of Eq. (39.7) yields

$$\iint (e + \frac{V^2}{2}) \rho V dA = \dot{m}\left[(e_2 + \frac{V_2^2}{2}) - (e_1 + \frac{V_1^2}{2}) \right]$$

The work done on the control surfaces is

$$\iint V(\tau . dA) = V_1 p_1 A_1 - V_2 p_2 A_2$$

The rate of heat transfer to the control volume is

$$- \iint q . dA = Q\dot{m}$$

where Q is the heat added per unit mass (in J/kg).

- Invoking all the aforesaid relations in Eq. (39.7) and dividing by $\dot{m}$, we get

$$e_2 + \frac{V_2^2}{2} - e_1 - \frac{V_1^2}{2} = -\frac{1}{\dot{m}}\left[p_2 V_2 A_2 - p_1 V_1 A_1 \right] + Q$$

We know that the density ρ is given by m/VA, hence the first term on the right may be expressed in terms of $\forall$ (specific volume=1/ρ).
Equation (39.8) can be rewritten as

$$e_2 + \frac{V_2^2}{2} - e_1 - \frac{V_1^2}{2} = p_1 \forall_1 - p_2 \forall_2 + Q$$

- NOTE:-

- $p_1 \forall_1$ is the work done (per unit mass) by the surrounding in pushing fluid into the control volume. Following a similar argument, is the $p_2 \forall_2$ work done by the fluid inside the control volume on the surroundings in pushing fluid out of the control volume.

Since

$$h = e + p\forall$$

Eq above gets reduced to

$$h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2} + Q$$

This is **energy equation**, which is valid even in the presence of friction or non-equilibrium conditions between secs 1 and 2.

- It is evident that **the sum of enthalpy and kinetic energy remains constant in an adiabatic flow**. Enthalpy performs a similar role that internal energy performs in a nonflowing system. The difference between the two types of systems is the flow work p $\forall$ required to push the fluid through a section

Bernoulli and Euler Equations

- For inviscid flows, the steady form of the momentum equation is the Euler equation,

$$\frac{dp}{\rho} + VdV = 0$$

Integrating along a streamline, we get the Bernoulli's equation for a compressible flow as

$$\int \frac{dp}{\rho} + \frac{V^2}{2} = const$$

- For adiabatic frictionless flows the Bernoulli's equation is identical to the energy equation. Recall, that this is an isentropic flow, so that the **Tds equation** is given by

$$Tds = dh - vdp$$

For isentropic flow, ds=0

Therefore,

$$dh = \frac{dp}{\rho}$$

Hence, the Euler equation can also be written as

$$VdV + dh = 0$$

Momentum Principle for a Control Volume

For a finite control volume between Sections 1 and 2 (Fig. 39.3), the momentum principle is

$$\begin{aligned} p_1 A_1 + p_2 A_2 + F &= \dot{m} V_2 - \dot{m} V_1 \\ &= \rho_2 V_2^2 A_2 - \rho_1 V_1^2 A_1 \end{aligned} \qquad (39.13)$$

where F is the x-component of resultant force exerted on the fluid by the walls. Note that the momentum principle, Eq. (39.13), is applicable even when **there are frictional dissipative**

processes within the control volume.

Bernoulli and Euler Equations

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$$\int \frac{dp}{\rho} + \frac{V^2}{2} = const$$

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$$Tds = dh - vdp$$

For isentropic flow, ds=0

Therefore,

$$dh = \frac{dp}{\rho}$$

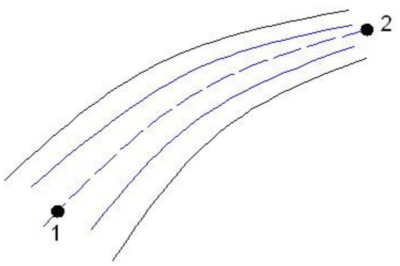
Hence, the Euler equation (39.11) can also be written as

$$VdV + dh = 0$$

This is identical to the adiabatic form of the energy Eq.

Bernoulli's Equation :

Bernoulli's theorem is a means of expressing the conservation of energy to the flow of fluids in a conduit. The total energy at any point, above arbitrary horizontal datum plane, is always constant value. It may be written



Two points joined by a streamline

$$\begin{matrix}\text{Pressure} \\ \text{energy per} \\ \text{unit weight}\end{matrix} + \begin{matrix}\text{Kinetic} \\ \text{energy per} \\ \text{unit weight}\end{matrix} + \begin{matrix}\text{Potential} \\ \text{energy per} \\ \text{unit weight}\end{matrix} = \begin{matrix}\text{Total} \\ \text{energy per} \\ \text{unit weight}\end{matrix}$$

$$\frac{p}{\rho g} + \frac{u^2}{2g} + z = H$$

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

$$\Pr essure\ Head = \frac{p}{\rho g}$$

$$Velocity\ \ head = \frac{u^2}{2g}$$

Potential head = z

Total head = H

By the principle of conservation of energy the total *energy* in the system does not change, Thus the total *head* does not change. The previous equation assumes no energy losses (e.g. from friction) or energy gains (e.g. from a pump) along the streamline. In practical applications this ideal situation does not exist, so the equation can be amended to include these, simply, by adding the appropriate energy terms:

$$\begin{array}{c}\text{Total}\\ \text{energy per}\\ \text{unit weight at 1}\end{array} = \begin{array}{c}\text{Total}\\ \text{energy per unit}\\ \text{weight at 2}\end{array} + \begin{array}{c}\text{Loss}\\ \text{per unit}\\ \text{weight}\end{array} + \begin{array}{c}\text{Work done}\\ \text{per unit}\\ \text{weight}\end{array} - \begin{array}{c}\text{Energy}\\ \text{supplied}\\ \text{per unit weight}\end{array}$$

$$\frac{p_2}{\rho g} + \frac{u_1^{\ 2}}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^{\ 2}}{2g} + z_2 + h_l + w - q$$

h_l : head loss.

w : work added to stream line by unit weight.

q : heat added to stream line by unit weight.

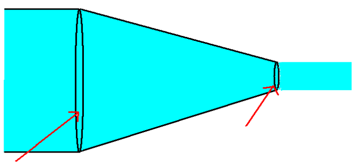
The Continuity Equation

Conservation of Mass:

In a confined system, all of the mass that enters the system, must also exit the system at the same time.

Flow rate = Q = Area x Velocity

$\rho_1 A_1 V_1$(mass inflow rate) = $\rho_2 A_2 V_2$(mass outflow rate)



If the fluid at both points is the same, then the density drops out, and you get the continuity equation:

$$A_1V_1 = A_2V_2$$

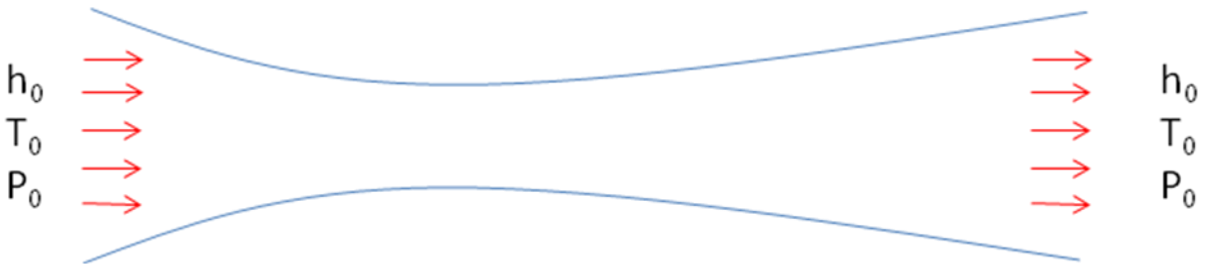
Therefore

If $A_2 < A_1$ then $V_2 > V_1$

Thus, water exiting a nozzle has a higher velocity

Isentropic Flow

- Adiabatic and Reversible
- No energy added, No energy losses
- Small an gradual change in flow variables



ds=0

An isentropic flow is a flow that is both adiabatic and reversible, that is no energy is added to the flow, and no energy losses occur due to friction or dissipative effects

Isentropic flows occur when the change in flow variables is small and gradual, such as the ideal flow through the nozzle shown below. Considering flow through a tube, as shown in the figure, if the flow is very gradually compressed (area decreases) and then gradually expanded (area increases), the flow conditions return to their original values. We say that such a process is reversible. From a consideration of the second law of thermodynamics, a reversible flow maintains a constant value of entropy. Engineers call this type of flow an isentropic flow.

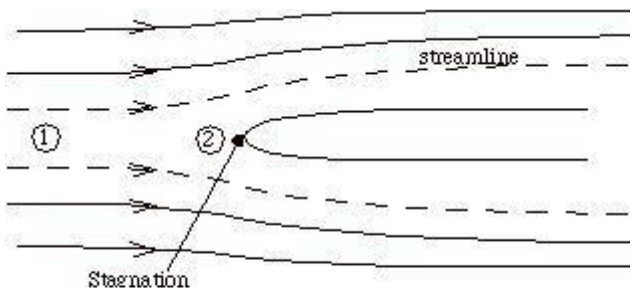
As a gas is forced through a tube, the gas molecules are deflected by the walls of the tube. If the speed of the gas is much less than the speed of sound of the gas, the density of the gas remains constant and the velocity of the flow increases. However, as the speed of the flow approaches the speed of sound we must consider compressibility effects on the gas. The density of the gas varies from one location to the next.

The generation of sound waves is an isentropic process. A supersonic flow that is turned while the flow area increases is also isentropic. We call this an isentropic expansion because of the area increase. If a supersonic flow is turned abruptly and the flow area decreases, shock waves are generated and the flow is irreversible. The isentropic relations are no longer valid and the flow is governed by the oblique or normal shock relations.

On this slide we have collected many of the important equations which describe an isentropic flow. We begin with the definition of the Mach number since this parameter appears in many of the isentropic flow equations. The Mach number M is the ratio of the speed of the flow c to the speed of sound a.

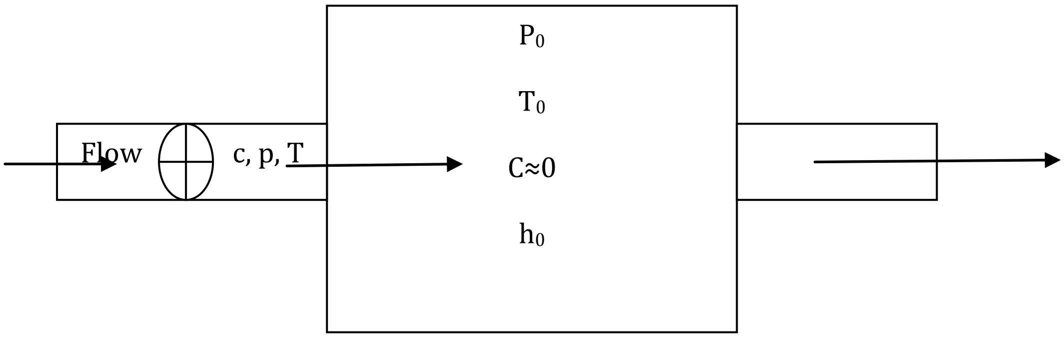
Stagnation Enthalpy

Suppose that our steady flow control volume is a set of streamlines describing the flow up to the nose of a blunt object, as in Figure



Streamlines and a stagnation region; a control volume can be drawn between the dashed streamlines and points 1 and 2

The streamlines are stationary in space, so there is no external work done on the fluid as it flows. If there is also no heat transferred to the flow (adiabatic),



Large Settling tank

Fig: Deceleration of gas to Stagnation state

Stagnation enthalpy

Stagnation enthalpy of a gas or vapor is its enthalpy when it is adiabatically decelerated to zero velocity at zero elevation.

Put

$h_1=h \quad c_1=c$ for the initial state

$h_2=h0 \quad c_2=0$ for the final state

We have the energy equation for a nozzle and diffuser is

$$h_1+\frac{1}{2}C_1^2=h_2+\frac{1}{2}C_2^2$$

By substituting this in above equation

We get

$$\boxed{h+\frac{1}{2}C^2=h_0}$$

Where

h_0 = Stagnation enthalpy

h = Static enthalpy

c = Fluid velocity m/s

In an adiabatic energy transformation process the stagnation enthalpy remain constant.

Stagnation Temperature (or) Total temperature (T_0)

Stagnation temperature of a gas when its isentropic ally decelerated to zero velocity at zero elevation

We know that

Stagnation enthalpy

$$h + \frac{1}{2}C^2 = h_0$$

We have stagnation enthalpy and static enthalpy for a perfect gas is

$$\boxed{\begin{aligned} h_0 &= c_p T_0 \\ h &= c_p T \end{aligned}}$$

By substituting this in above equation

We get

$$C_p T_0 = C_p T + \frac{C^2}{2}$$

Divide by Cp through out the eqn.

$$T_0 = T + \frac{C^2}{2C_p}$$

Where

To	=	stagnation temperature
T	=	static temperature

$$\frac{c^2}{2C_p}$$ = dynamic temperature or velocity temperature or temperature equivalent to stream velocity (Tc)

$C^2/2C_p$ = dynamic temperature

(Amount of rising temp. when the fluid is brought to stagnation adiabatically)

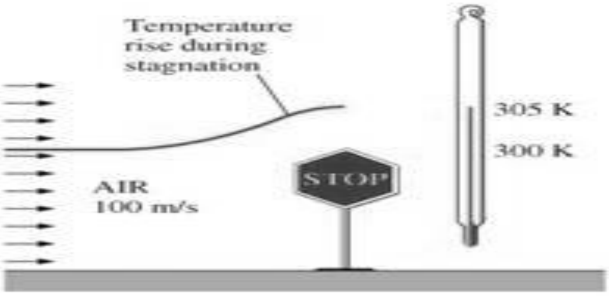


FIGURE 17–5
The temperature of an ideal gas flowing at a velocity V rises by $V^2/2c_p$ when it is brought to a complete stop.

Stagnation temperature, $T_0 = T + \frac{C^2}{2C_p}$

Divided by T

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{C^2}{2C_pT}$$

We know that

$$C_p = \frac{\gamma R}{\gamma - 1}$$

$$a = \sqrt{\gamma RT}$$

M=C/a

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{C^2}{2\frac{\gamma RT}{\gamma - 1}}$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{C^2}{2\frac{a^2}{\gamma - 1}}$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} \times \frac{C^2}{a^2}$$

$$where \frac{C^2}{a^2} = M^2$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} \times M^2$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} \times M^2$$

Where

To = stagnation temperature

T = static temperature

M = Mach number (C/a)

Stagnation Pressure, [Po] or total pressure

Stagnation pressure of a gas when it is isentropically decelerated to zero velocity at zero elevation

For isentropic flow

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}}$$

For stagnation condition

Put

$$P_2 = P_0$$

$$T_2 = T_0$$

$$P_1 = P$$

$$T_1 = T$$

$$\Rightarrow \frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \frac{P_0}{P} = \left[1+\frac{\gamma-1}{2}\times M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_0}{P} = \left[1+\frac{\gamma-1}{2}\times M^2\right]^{\frac{\gamma}{\gamma-1}}$$

Where

P_0 = stagnation Pressure

P = Static pressure

M = Mach number (C/a)

Stagnation velocity of sound[a_0]

We know that the acoustic velocity of sound

$a = \sqrt{\gamma RT}$.

For a given value of Stagnation temperature, Stagnation velocity of sound

For stagnation condition

Put

$$a = a_0$$

$$T = T_0$$

$$a_0 = \sqrt{\gamma R T_0}$$

$$a_0 = \sqrt{\frac{C_p(\gamma-1)\gamma T_0}{\gamma}}$$

Where

$$R = \frac{C_p(\gamma-1)}{\gamma}$$

$$a_0 = \sqrt{C_p(\gamma-1)T_0}$$

Where

$$h_0 = C_p T_0$$

$$a_0 = \sqrt{h_0(\gamma-1)}$$

Show that $\dfrac{\rho_0}{\rho} = \left[1+\dfrac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}$

We have

$P = \rho RT$

$Po = \rho_o RT_o$

Dividing

$$\frac{p_o}{p} = \frac{\rho_0}{\rho} = \frac{T_o}{T}$$

$$\therefore \frac{\rho_0}{\rho} = \frac{p_o/p}{T_o/T}$$

$$= \frac{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M^2\right]}$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{1}{\gamma - 1}}$$

Or

For isentropic flow

$$\frac{\rho}{\rho_0} = \left(\frac{P_0}{P_1}\right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma-1}}$$

Various regions of flow

The adiabatic energy equation for a perfect gas is derived in terms of velocity of fluid (C) and Velocity of sound[a_0]

We have stagnation enthalpy and static enthalpy for a perfect gas is

$$h_0 = c_p T_0$$

$$h = c_p T$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

$$a = \sqrt{\gamma R T}$$

$$h + \frac{1}{2} C^2 = h_0$$

Put

$$a = a_0$$

$$T = T_0$$

$$a_0 = \sqrt{\gamma \mathrm{R} \mathrm{T}_0}$$

$$a_0 = \sqrt{\frac{\mathrm{C_p}(\gamma - 1)\gamma \mathrm{T_0}}{\gamma}}$$

Where

$$\mathrm{R} = \frac{\gamma \mathrm{C_P T}}{\gamma - 1}$$

$$a = \sqrt{\gamma \mathrm{RT}}$$

$$a^2 = \gamma \mathrm{RT}$$

$$\mathrm{h} = \frac{a^2}{\gamma - 1}$$

$$h + \frac{1}{2}C^2 = h_0$$

$$\Rightarrow \quad h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2}C^2$$

Case

$$At$$
$$T = 0,$$
$$h = o$$

Put

$$c = c_{\max}$$

$$h_0 = \mathrm{C_p T_0} + \frac{1}{2}C^2{}_{\max}$$

$$h_0 = \frac{1}{2} C^2{}_{max}$$

Put

at

$C = 0$

$a = a_0$

$h_0 = h$

$$h_0 = \frac{a_0{}^2}{\gamma - 1}$$

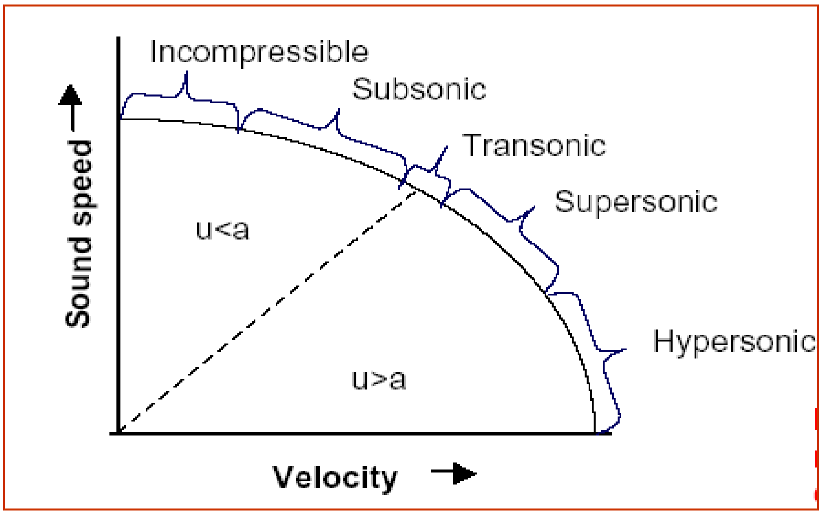
$$h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} C^2 = \frac{a_0{}^2}{\gamma - 1} = \frac{1}{2} C^2{}_{max}$$

$$\boxed{h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} C^2 = \frac{a_0{}^2}{\gamma - 1} = \frac{1}{2} C^2{}_{max}}$$

Flow Regime Classification

- **Subsonic Flow**

  0.8 < M_∞



- **Transonic Flow**

  0.8 > M_∞ > 1.2

- **Supersonic Flow**

  M_∞ > 1.2

- **Hypersonic Flow**

  M_∞ > 5

1. <u>Incompressible region</u>

In incompressible flow region fluid velocity (c) is much smaller than the sound velocity

(a). Therefore the Mach number (M = c/a) is very low.

Eg: flow through nozzles

2. <u>Subsonic flow region</u>

The subsonic flow region is on the right of the incompressible flow region. In subsonic flow, fluid velocity (c) is less than the sound velocity (a) and the mach number in this region is always less than unity.

i.e. $m = c/a < 1$.

Eg: passenger air craft

3. Sonic flow region

If the fluid velocity (c) is equal to the sound velocity (a), that type of flow is known as sonic flow .In sonic flow Mach number value is unity.

$M = c/a = 1 \Rightarrow c = a$.

Eg: nozzle throat

4. Transonic flow region

If the fluid velocity close to the speed of sound, that type of flow is known as transonic flow .In transonic flow, Mach number value is in between 0.8 and 1.2. i.e.$0.8 < M < 1.2$.

5. Supersonic flow region

The supersonic region is in the right of the transonic flow region. In supersonic flow, fluid velocity (c) is more than the sound velocity (a) and the Mach number in this region is always greater than unity.

i.e.$M = c/a > 1$.Eg: military air crafts

6. Hypersonic flow region

In hypersonic flow region, fluid velocity (c) is much greater than sound velocity (a).In this flow, Mach number value is always greater than 5.

i.e. $M = c/a > 5$. Eg: rockets

REFERENCE VELOCITIES

In compressible flow analysis it is often convenient to express fluid velocities is non dimensional forms**.**

1. Local velocity of sound

2. Stagnation velocity of sound

3. Maximum velocity of fluid

4. Critical velocity of fluid/sound. $C^*=a^*$

Maximum velocity of fluid

From adiabatic energy equation has two components of the total energy: the enthalpy h and the kinetic energy $\frac{1}{2}C^2$.If kinetic energy is absent the total energy is entirely energy represented by the stagnation enthalpy h_0 .the other extreme conditions which can be conceived is when the entire energy is made up of kinetic energy only=0 and C = Cmax.The fluid velocity (Cmax) corresponding to this condition is the maximum velocity that would be achived by the fluid when it is accelerated to absoulute zero temperature (T = 0 ,T = 0)in an imaginary adiabatic expansion process.

$$C\max = \sqrt{2h_0}$$

For a perfect gas

$$C\max = \sqrt{2C_pT_0}$$

$$= \sqrt{\frac{2\gamma}{\gamma-1}RT_0}$$

Equation $h_0 = \frac{a^2}{\gamma-1} + \frac{1}{2}C^2 = \frac{a_0^{\ 2}}{\gamma-1} = \frac{1}{2}C^2_{\max}$ and equation

$$= \sqrt{\frac{2\gamma}{\gamma - 1} RT_0}$$ Yield

$$\frac{C_{max}}{a_0} = \sqrt{\frac{2}{\gamma - 1}} = 2.24\left(for\gamma = 1.4\right)$$

Critical velocity of sound

It is the velocity of flow that would exist if the flow is isentropically accelerated or decelerated to unit Mach number (critical condition)

We have

Considering the *section (where M = 1) and its stagnation section

We have

$M \Rightarrow 1$

$T \Rightarrow T*$

$p \Rightarrow p*$

$\rho \Rightarrow \rho*$

$M_{critical} = \frac{C^*}{a^*}$

$$\Rightarrow \frac{T_0}{T^*} = \left[\frac{\gamma + 1}{2} \right]$$

Multiplying both sides with γ R

$$T_0 = T^* \left[\frac{\gamma + 1}{2} \right]$$

$$a^2{}_0 = a^{*2} \times \frac{\gamma + 1}{2}$$

$$\frac{a^*}{a_0} = \frac{C^*}{a_0} = \sqrt{\frac{2}{\gamma + 1}}$$

From Equations

$$\frac{C_{\max}}{a_0} = \sqrt{\frac{2}{\gamma - 1}} \text{ and } \frac{a^*}{a_0} = \frac{C^*}{a_0} = \sqrt{\frac{2}{\gamma + 1}}$$

$\frac{C_{\max}}{a^*} = \frac{C_{\max}}{a^*} = \sqrt{\frac{\gamma + 1}{\gamma - 1}} = 2.45(for\gamma)$ Finally we can written the equation is

$$C_{\max} = a^* \times \sqrt{\frac{\gamma + 1}{\gamma - 1}}$$

$$= C^2{}_{\max} = a^{2*} \times \frac{\gamma + 1}{\gamma - 1}$$

$$h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} C^2 = \frac{1}{2} C^2{}_{\max} = \frac{1}{2} a^{2*} \times \frac{\gamma + 1}{\gamma - 1}$$

Expressions for $\frac{T_o}{T*}, \frac{p_o}{p*}$ and $\frac{\rho_o}{\rho*}$

Considering the *section (where M = 1) and its stagnation section

We have

$M \Rightarrow 1$

$T \Rightarrow T*$

$p \Rightarrow p*$

$\rho \Rightarrow \rho^*$

We already have

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\therefore \frac{T_o}{T^*} = \frac{\gamma + 1}{2}$$

Similarly

$$\frac{p_o}{p^*} = \left[1 + \frac{\gamma - 1}{2}\right]^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{p_o}{p^*} = \left[1 + \frac{\gamma + 1}{2}\right]^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho_o}{\rho^*} = \left[1 + \frac{\gamma - 1}{2}\right]^{\frac{1}{\gamma - 1}}$$

$$= \left[\frac{\gamma - 1}{2}\right]^{\frac{1}{\gamma - 1}}$$

$$\frac{\rho_o}{\rho^*} = \left[\frac{\gamma + 1}{2}\right]^{\frac{1}{\gamma - 1}}$$

Expressions for $\frac{T}{T^*}, \frac{p}{p^*}$ and $\frac{\rho}{\rho^*}$

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

At $M = 1 \quad T = T^*$

$$\therefore \frac{T_o}{T^*} = \frac{\gamma + 1}{2}$$

$$\frac{T}{T^*} = \frac{T_o / T^*}{T_o / T}$$

$$\frac{T}{T^*} = \frac{\frac{\gamma+1}{2}}{\left[1+\frac{\gamma-1}{2}M^2\right]}$$

$$\frac{p}{p^*} = \frac{p_o/p^*}{p_o/p} \qquad \frac{\left(\frac{r+1}{2}\right)^{r/r-1}}{\left[1+\frac{r-1}{2}M^2\right]^{r/r-1}}$$

$$= \left[\left[\frac{r+1}{2}+\frac{r+1}{2}\times\frac{2}{r-1}m^2\right]^{r/r-1}\right.$$

$$\frac{p}{p^*} = \frac{\left[\frac{\gamma+1}{2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{\rho}{\rho^*} = \frac{\rho_o/\rho^*}{\rho_o/\rho}$$

$$\frac{\rho}{\rho^*} = \frac{\left[\frac{\gamma+1}{2}\right]^{\frac{1}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{1}{\gamma-1}}}$$

Reference Mach number M^*

In the analysis of high speed flows, another Mach number called $M^*$is employed .It is defined as the non dymensionlizing the fluid velocity by the critical fluid velocity or the sound velocity.

That is,

$$M^* = \frac{C}{a^*} = \frac{C}{C^*}$$

$$M^{*2} = \frac{C^2}{a^{*2}} = \frac{C}{a^2} \times \frac{a^2}{a^{*2}} = M^2 \frac{a^2}{a^{*2}}$$

Some it is more convenient to use **M^* instead of M because**

(i) at high fluid velocities M approaches infinity

(ii) M is not proportional to the velocity alone

It should be pointed out here that M^*does not mean M = 1this only other type of Mach number.

We have $h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2}C^2 = \frac{1}{2}C^2{}_{\max} = \frac{1}{2}a^{2*} \times \frac{\gamma + 1}{\gamma - 1}$

= $\frac{1}{2}a^{2*} \times \frac{\gamma + 1}{\gamma - 1}$ =

Multiplying by 2

$$\frac{2a^2}{\gamma - 1} + C^2 = \frac{\gamma + 1}{\gamma - 1} a * 2$$

Divided by $a * 2$

$$\frac{C^2}{a^{*2}} + \frac{2}{\gamma - 1}\frac{a^2}{a *^2} = \frac{\gamma + 1}{\gamma - 1}$$

$$M^{*2} + \frac{2}{\gamma - 1}\frac{M^{*2}}{M^2} = \frac{\gamma + 1}{\gamma - 1}$$

$\div$by $\frac{\gamma-1}{2}$

$$M^{*2}\left(1+\frac{2}{\gamma-1}\frac{1}{M^2}\right)=\frac{\gamma+1}{\gamma-1}$$

$$M^{*2}\left(\frac{2}{\gamma-1}\right)\left(\frac{\gamma-1}{2}+\frac{1}{M^2}\right)=\frac{\gamma+1}{\gamma-1}$$

$$M^{*2}=\frac{\frac{1}{2}(\gamma+1)M^2}{1+\frac{1}{2}(\gamma-1)M^2}$$

When $M^*=0$ at $M = o$

$M^* = 1$ at $M = 1$

Eqn.we have

$$M^{*2}+\frac{2}{\gamma-1}\frac{M^{*2}}{M^2}=\frac{\gamma+1}{\gamma-1}$$

It gives

$$\frac{2}{\gamma-1}\frac{M^{*2}}{M^2}=\frac{\gamma+1}{\gamma-1}-M^{*2}$$

$$\frac{M^{*2}}{M^2}=\frac{\gamma+1}{2}-\frac{\gamma-1}{2}M^{*2}$$

$$M^2 = \frac{\left\{\frac{2}{\gamma+1}\right\}M^{*2}}{1-\left\{\frac{\gamma+1}{\gamma-1}\right\}M^{*2}}$$

$$1-\left\{\frac{\gamma+1}{\gamma-1}\right\}M^{*2} = 0$$

$$M^{*}{}_{\max} \frac{C_{\max}}{C^{*}} = \sqrt{\frac{\gamma+1}{\gamma-1}} = 2.45(\mathit{for}\gamma = 1.4)$$

$$C_r = \frac{C}{C_{\max}.}$$

At M = ∝

$$1-\left\{\frac{\gamma+1}{\gamma-1}\right\}M^{*2} = 0$$

$$M^{*}{}_{\max} \frac{C_{\max}}{C^{*}} = \sqrt{\frac{\gamma+1}{\gamma-1}} = 2.45(\mathit{for}\gamma = 1.4)$$

Crocco number

A non -dimensional fluid velocity can be defined by using the Maximum fluid velocity C_{max}

$$C_r = \frac{\mathit{flowvelocity}}{\max.\mathit{fluidvelocity}}$$

$$C_r = \frac{C}{C_{\max}.}$$

Q22) Derive Bernoulli's equation for compressible flow in the form

$$\frac{V^2}{2}+\frac{K}{K-1}\frac{p_0}{\rho_0}\left(\frac{P}{P_o}\right)^{\frac{K}{K-1}}=\frac{K}{K-1}\left(\frac{P_0}{\rho_0}\right)$$

Ans:

Euler's equation, $\frac{dP}{\rho}+\gamma dv+gdz=0$

Neglecting potential head we have

$$\frac{dP}{\rho}+vdv=0$$

Integrating we have

$$\frac{K}{K-1}\frac{P}{\rho}+\frac{V^2}{2} = \text{constant}$$

Applying this equation to static and stagnation of the flowing fluid

$$\frac{K}{K-1}\frac{P}{\rho}+\frac{V^2}{2}=\frac{K}{(K-1)}\frac{P}{\rho_o} \qquad (1)$$

ie, $\frac{V^2}{2}+\frac{K}{K-1}\frac{P}{\rho}=\frac{K}{(K-1)}\frac{P}{\rho_o}$

We have to prove

$$\frac{P}{\rho}=\frac{P_o}{\rho_o}\left(\frac{P}{P_o}\right)^{\frac{K}{K-1}}$$

$$\frac{P}{\rho}=\frac{P_o}{\rho_o}\cdot\frac{P}{\frac{P_o}{\rho_o}\times\rho} = \frac{P_o}{\rho_o}\cdot\left[\frac{P}{\frac{P_o}{\rho_o}\times\rho}\right] = \frac{P_o}{\rho_o}\cdot\left[\frac{P}{P_o}\cdot\frac{\rho_o}{\rho}\right]$$

$$=\frac{P_o}{\rho_o}\cdot\left[\frac{P}{P_o}\left(\frac{\rho_o}{\rho}\right)^{\frac{1}{K}}\right] = \frac{P_o}{\rho_o}\cdot\left[\frac{P}{P_o}\left(\frac{P_o}{P}\right)^{\frac{1}{K}}\right] = \frac{P_o}{\rho_o}\left(\frac{P}{P_o}\right)^{\frac{K-1}{K}}$$

Substituting in (1), we have

$$\frac{V^2}{2}+\frac{K}{K-1}\ \frac{P_o}{\rho_o}\left(\frac{P}{P_o}\right)^{\frac{K}{K-1}}=\frac{K}{(K-1)}\ \frac{P_o}{\rho_o}$$

Q23) The dynamic pressure ½rV^2 is the difference between stagnation and static pressures for incompressible, but not for compressible flow explain why?

Ans: For incompressible flow

$$\frac{dP}{\rho}+vdv=0$$

$$\frac{P}{\rho}+\frac{V^2}{2}=\text{constant}$$

$$\frac{P}{\rho}+\frac{V^2}{2}=\frac{P_o}{\rho}$$

$$P_o-P=½rV^2$$

For compressible flow

$$\frac{K}{K-1}\ \frac{P}{\rho}+\frac{V^2}{2}=\frac{K}{K-1}\ \frac{P_o}{\rho_o}$$

We have

$$\frac{P_o}{P}=1+\frac{K}{K-1}\ \frac{\frac{K-1}{2}M^2}{1\,!}+\frac{K}{K-1}\left(\frac{K}{K-1}-1\right)\frac{\left(\frac{K-1}{2}\ M^2\right)^2}{2\,!}+$$

$$\frac{K}{K-1}\left(\frac{K}{K-1}-1\right)\left(\frac{K}{K-1}-2\right)\frac{\left(\frac{K-1}{2}\ M^2\right)^3}{3\,!}+......$$

$$\frac{P_o}{P}=1+\frac{K}{2}M^2+\frac{K}{8}M^4+\frac{K.1.(2-K)}{8\times 6}M^6+......$$

$$\frac{P_o}{P}-1=\frac{K}{2}M^2\left(1+\frac{M^2}{4}+\frac{(2-K)M^4}{24}+.....\right)$$

$$P_o - P = P\frac{K}{2}M^2\left(1+\frac{M^2}{4}+\frac{(2-K)M^4}{24}+........\right)$$

$$P_o - P = \frac{\rho RTK}{2}M^2\left(1+\frac{M^2}{4}+\frac{(2-K)}{24}M^4+.......\right)$$

$$=\frac{\rho}{2}C^2M^2\left(1+\frac{M^2}{4}+\frac{(2-K)}{24}M^4+.........\right)$$

$$P_o - P = \frac{1}{2}\rho V^2\left(1+\frac{M^2}{4}+\frac{(2-K)}{24}M^4+.........\right)$$

Thus dynamic pressure ½rV^2 is not the difference between stagnation and static pressures for the compressible flow.

Q 24) What is the effect of mach number on compressibility. Show that for sonic flow the deviation between compressible and non compressible values of pressure co-efficient $\frac{P_o - P}{½\rho V^2}$ of a perfect gas is about 27.5%.

Ans: We have

$$P_o - P = \frac{1}{2}\rho V^2\left(1+\frac{M^2}{4}+\frac{(2-K)}{24}M^4+........\right)$$

ie, $\frac{P_o - P}{½\rho V^2} = \left[1+\frac{M^2}{4}+\frac{2-K}{24}M^4+.....\right]$

For sonic flow M = 1, Hence (Assume K = 1.4)

Compressible flow

$$\frac{P_o - P}{½\rho V^2} = \left[1+\frac{1}{4}+\frac{2-1.4}{24}+.......\right] = \left[\frac{5}{4}+\frac{0.6}{24}\right] = \left[\frac{5}{4}+\frac{1}{40}\right] = \frac{50+1}{40} = \frac{51}{40}$$

Incompressible flow $\frac{P_o - P}{½\rho V^2} = 1$

Deviation = $\left[\frac{51}{40}-1\right] = \frac{11}{40} = 0.275 = 27.5\%$

Q25) Starting from continuity and momentum, obtain

$$\frac{P_o}{P} = \left[1 + \frac{K-1}{2}M^2\right]^{\frac{K}{K-1}}$$

Ans: Newton's Law

$$\sum F_{xx} = \dot{m}_{out} V_{xx\,out} - \dot{m}_{in}\ V_{xxin}$$

By continuity $\dot{m}_{out} = \dot{m}_{in}$

$$\therefore\ \Sigma F_{xx} = \dot{m}\left[V_{xx\ out} - V_{xx\ in}\right]$$

Finally we get,

$$\frac{dP}{\rho} + VdV = 0$$

Integrating this for compressible,

$$\frac{K}{K-1}\frac{P}{\rho} + \frac{V^2}{2} \ = \text{constant}$$

Applying this to static and stagnation state of flowing fluid

$$\frac{K}{K-1}\frac{P}{\rho} + \frac{V^2}{2} = \frac{K}{K-1}\ \frac{P_o}{\rho_o}$$

$$\frac{K}{K-1}\left[\frac{P_o}{\rho_o} - \frac{P}{\rho}\right] = \frac{V^2}{2}$$

$$\left[\frac{P_o}{\rho_o} - \frac{P}{\rho}\right] = \frac{K-1}{2K}V^2$$

$$\frac{P}{\rho}\left[\frac{P_o}{\rho}\ \frac{\rho}{P} - 1\right] = \frac{K-1}{2K}V^2$$

$$RT\left[\frac{P_o}{P}\left(\frac{P}{P_o}\right)^{\frac{1}{K}} - 1\right] = \frac{K-1}{2K}V^2$$

$$RT\left[\frac{P_o}{P}\left(\frac{P_o}{P}\right)^{\frac{-1}{K}}-1\right]=\frac{K-1}{2C^2}\,V^2 \qquad \left[\because\ KRT=C^2\right]$$

$$\left(\frac{P_o}{P}\right)^{\frac{K-1}{K}}-1=\frac{K-1}{2}M^2$$

$$M^*=M\left[\frac{\frac{K+1}{2}}{1+\frac{K-1}{2}M^2}\right]$$

Q28) Prove that $\frac{dT}{T}=\left(1-K\right)M^2\,\frac{dV}{V}$

Ans:

Energy Equation:

$$h+\frac{V^2}{2}=h_o$$

$$C_pT+\frac{V^2}{2}=C_pT_o$$

$$C_pdT+VdV=0$$

$$dT=\frac{-VdV}{KR}\left(K-1\right)$$

$$\frac{dT}{T}=\frac{-VdV\left(K-1\right)}{KRT}=\frac{\left(1-K\right)V^2}{C^2}\,\frac{dV}{V}$$

$$\frac{dT}{T}=\left(1-K\right)M^2\,\frac{dV}{V}$$

Q29) Prove the following

(i) $\frac{V_{max}}{C_o}=\sqrt{\frac{2}{K-1}}$ (ii) $\frac{C^*}{C_o}=\sqrt{\frac{2}{K+1}}$

(iii) $\frac{V_{max}}{C^*}=\sqrt{\frac{K+1}{K-1}}$ (iv) $C_{max}=\sqrt{C^2+\frac{2a^2}{K-1}}$

(v) $C^* = \sqrt{\dfrac{2C^2 + V^2(K-1)}{K+1}}$ (vi) $V_{max} = \sqrt{V^2 + \dfrac{2C^2}{K-1}}$

Ans:

Energy equation

$$h + \frac{V^2}{2} = h_o$$

$$\frac{V^2}{2} = (h_o - h)$$

$$V^2 = 2C_p(T_o - T)$$

For maximum V, T $\rightarrow$ 0

$$V_{max}^2 = 2C_pT_o$$

$$C_o^2 = KRT_o$$

$$\frac{V_{max}^2}{C_0^2} = \frac{2C_pT_o}{KRT_o} = \frac{2KA}{(K-1)KR} = \frac{2}{(K-1)}$$

$$\frac{V_{max}}{C_o} = \sqrt{\frac{2}{(K-1)}}$$

(ii) $C^* = \sqrt{KRT^*}$

$$C_o = \sqrt{KRT_o}$$

$$\frac{C^*}{C_0} = \sqrt{\frac{T^*}{T_o}}$$

$$\frac{C^*}{C} = \sqrt{\frac{2}{(K+1)}}$$

(iii) $V_{max} = \sqrt{2C_pT_o}$

$$\frac{P_o}{P} = \left[1 + \frac{K-1}{2} M^2\right]^{\frac{K}{K-1}}$$

Q26) Show that velocity of sound can be expressed in the form $C^2 = \frac{K}{\rho}$ where K = bulk modules of elasticity.

Ans:

Prove upto $C^2 = \left.\frac{dP}{d\rho}\right]_{S=constant}$

By definition

K = Bulk modules of elasticity = $\frac{\text{Increase in pressure}}{\text{Relative change in volume}} = \frac{dP}{-\left(\frac{dV}{V}\right)}$

But, $V = \frac{1}{\rho}$

$$dV = \frac{1}{\rho^2}\, d\rho$$

$$\frac{dV}{V} = \frac{-1}{\rho^2} \frac{d\rho}{1/\rho} = -\frac{d\rho}{\rho}$$

Hence, $K = \rho \frac{dP}{d\rho}$

$$\frac{dP}{d\rho} = \frac{K}{\rho}$$

$$c^2 = \left.\frac{dP}{d\rho}\right]_s = \frac{K}{\rho}$$

Q27) Define M* and show that $M^* = M\left[\frac{\frac{K+1}{2}}{1 + \frac{K-1}{2} M^2}\right]^{½}$

$M_1 = \frac{V_1}{C_1}$ both measured at the same section.

$M^* = \frac{V_1}{C^*}$ $\quad M^* = \frac{V}{C^*}$

Where V = flow velocity at given section. C* = sonic velocity where mach number = 1.

$$\left(M^*\right)^2 = \frac{V^2}{C^{*2}} = \frac{V^2}{C^2} \cdot \frac{C^2}{C^{*2}} \quad = M^2 \frac{T}{T^*}$$

$$\frac{T_o}{T} = \left[1 + \frac{K-1}{2} M^2\right]$$

$$\frac{T_o}{T^*} = \frac{K+1}{2}$$

$$\frac{T}{T^*} = \frac{T_o / T^*}{T_o / T} = \left[\frac{\frac{K+1}{2}}{1 + \frac{K-1}{2} M^2}\right]$$

$$\left(M^*\right)^2 = M^2 \left[\frac{\frac{K+1}{2}}{1 + \frac{K-1}{2} M^2}\right]$$

$$C^* = \sqrt{KRT^*}$$

$$\frac{V_{max}}{C^*} = \sqrt{\frac{2(KR)T_o}{(K-1)KRT^*}} \quad = \sqrt{\frac{2}{K-1} \frac{K+1}{2}}$$

$$\frac{V_{max}}{C^*} = \sqrt{\frac{K+1}{K-1}}$$

(iv) and (vi)

We have $h + \frac{V^2}{2} = h_o$

$$\frac{V^2}{2} = \left(h_o - h\right) = C_p \left(T_o - T\right)$$

For max value of $V, T \to 0$

ie, $V_{max}^2 = C_p T_o = h_o$

$$V_{max}^2 = 2h_o$$

$$V_{max}^2 = 2\left[h + \frac{V^2}{2}\right]$$

$$\frac{V_{max}^2}{2} = C_p T + \frac{V^2}{2} = \frac{KRT}{(K-1)} + \frac{V^2}{2}$$

$$\frac{V_{max}^2}{2} = \frac{C^2}{K-1} + \frac{V^2}{2}$$

$$V_{max}^2 = \frac{2C^2}{K-1} + V^2$$

$$V_{max} = \sqrt{V^2 + \frac{2C^2}{K-1}}$$

Changing $C \to a$ = sonic velocity and $V \to C$ = flow velocity.

$$C_{max} = \sqrt{C^2 + \frac{2a^2}{K-1}}$$

(v) $h + \frac{V^2}{2} = h_o$

$$C_p T + \frac{V^2}{2} = C_p T_o$$

$$\frac{C^2}{K-1} + \frac{V^2}{2} = \frac{C_o^2}{K-1}$$

$$\frac{C_o^2}{C^{*2}} = \frac{T_o}{T^*} = \frac{K+1}{2}$$

$$C_0^2 = \left(\frac{K+1}{2}\right) C^{*2}$$

$$\frac{C^2}{K-1}+\frac{V^2}{2}=\frac{C_o^2}{K-1}=\frac{\frac{(K+1)}{2}(C*)^2}{K-1}$$

$$\frac{(K+1)}{2}(C*)^2=C^2+\frac{V^2(K-1)}{2}$$

$$C^{*2}=\frac{2}{K+1}\left[C^2+\frac{V^2(K-1)}{2}\right]=\frac{2C^2}{K+1}+\frac{V^2(K-1)}{K+1}$$

$$C*=\sqrt{\left[\frac{2C^2V^2(K-1)}{K+1}\right]}$$

Q. Air at 20 ^{0} C and 250 m/s enters a diffuser. What will be Mach No. at diffuser. What will be Mach no. at diffuser inlet?

$T = 20^0\,C = 293K$

$C = 250m/s$

$$M=\frac{C}{a}$$

$$a=\sqrt{\gamma RT}$$

$$M=\frac{C}{\sqrt{\gamma RT}} : \frac{250}{\sqrt{1.4\times 287\times 293}}=0.72$$

M = 0.72 (FLOW IS SUBSONIC)

Q.) A supersonic aircraft flies at an altitude of 2000m, where the air temperature is 3^oC. Determine the speed of aircraft if its sound is heard 4.5 sec after it passes over an observer.

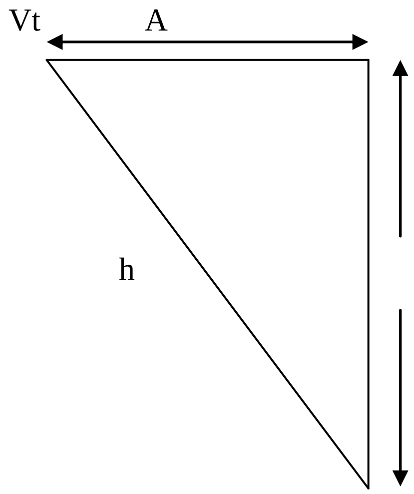
Solution:

The sonic speed is calculated from the relation:

$$a = \sqrt{\gamma RT}$$

$$= \sqrt{1.41 \times 287 \times 276}$$

$$= 334.2 \text{ m/s}$$



Here O = Observer

A = Position of aircraft when it is over the head of the observer.

B = Position of aircraft at the instant the observer fells the Disturbance due to aircraft.

The ground observer will feel the motion of the aircraft only when the Mach cone represented by the OB covers his position. If t is the time elapsed, the distance travelled by the aircraft between the referred instance is Vt and

$$\tan \alpha = \frac{h}{Vt} = \frac{2000}{4.5V} \qquad \text{...............(i)}$$

where h is the altitude of flight and V is the relative velocity between the observer and the aircraft.

From the relation, $\frac{V}{a} = M = \frac{1}{\sin \alpha}$, we have :

$$a = V \sin \alpha \; ; \; 334.2 = V \sin \alpha \qquad \text{..............(ii)}$$

From expression (i) and (ii),

$$\tan \alpha = \frac{2000 \times \sin \alpha}{4.5 \times 334.2} = 1.33 \sin \alpha \; ; \; \cos \alpha = \frac{1}{1.33} = .752$$

Hence

$$\alpha = 41.24^{o} \text{ and } \sin \alpha = .6592$$

Therefore

Velocity of aircraft = $\frac{334.2}{.6592}$ = 506.98 m/s

Mach number M = $\frac{1}{\sin\alpha} = \frac{1}{.6592}$ = 1.517

Module - I (Problems)

Problems

Q1) Atm air at 1.1×10^5 N/m^2 and 65^{o}C is accelerated isentropically to a mach number of 1. Find final temp, pressure and flow velocity.

Ans.

Assume, the given state is the stagnation state.

For isentropic flow,

$$\frac{T_o}{T} = f(M) = 1+\frac{K-1}{2}M^2$$ Fig.

From data book, is entropic table, K = 1.4

$$\left.\frac{T}{T_o}\right]_{M=1} = 0.834$$

T = T* = 8.834 × T_o

= 0.834 × 338

T = 281.9 K

Final temperature T* = 281.9 K

$$\left.\frac{P}{P_o}\right]_{M=1} = 0.528$$

P = P* = P_o × 0.528

= 1.1 × 10^5 × 0.528

= 0.581 × 10^5 N/m^2

Final pressure P* = 0.581×10^5 N/m^2

$$M = \frac{V}{C}$$

Final mach number = 1

$$1 = \frac{V_{final}}{C_{final}} = \frac{V*}{C*}$$

$$V* = C* = \sqrt{KRT*}$$

$$= 20.05\sqrt{281.9}$$

Final velocity V* = 336.6 ms^{-1}

Q2) The pressure, temperature velocity of air at a point in a flow field are 1.2×10^5N/m^2, 37^{o}C and 250 ms^{-1} respectively. Find the stagnation pressure and stagnation temperature corresponding to given condition

Ans

Match number (M_1)

$$M_1 = \frac{V_1}{C_1} = \frac{c_1}{a_1}$$ Fig.

$$= \frac{250}{\sqrt{KRT_1}} = \frac{250}{20.05\sqrt{310}} = \frac{250}{353.02}$$

M_1 = 0.708

Stagnation properties (P_o, T_o)

From isentropic tables (K = 1.4)

$$\left.\frac{T_1}{T_o}\right]_{M_1=0.708} = 0.908$$

$$T_o = \frac{T_1}{0.908} = \frac{310}{0.908}$$

T_o = 341.41 K

Stagnation temperature T_o = 341.41 K

$$\left.\frac{P_1}{P_o}\right]_{M_1=0.708} = 0.714$$

$$P_o = \frac{P_1}{0.714} = \frac{1.2\times10^5}{0.714}$$

$= 1.681 \times 10^5$ N/m^2

Stagnation pressure, $P_o = 1.681 \times 10^5$ N/m^2

Q 3) Determine velocity of sound in air at 38^{o}C.

Velocity of sound C = $\sqrt{KRT}$

$= \sqrt{1.4\times287\times311}$

C = 353.5 ms^{-1}

Velocity of sound at 30^{o}C

= 353.5 ms^{-1}

Q 4) Air flows through a duct with a velocity 300 ms^{-1} pressure 1 bar, temp. 30^{o}C. Find (i) Stagnation pressure and temperature (ii) Velocity of sound in dynamic and stagnation condition

Ans.

Velocity of sound at (1)

$C_1 = \sqrt{KRT_1} = 20.05\ \sqrt{303}$ Fig.

C_1 =349 ms^{-1}

$$M_1 = \frac{V_1}{C_1} = \frac{300}{349}$$

$M_1 = 0.86$

Stagnation properties (P_o and T_o) from Tables

$$\left.\frac{T_1}{T_o}\right]_{M=0.86} = 0.871$$

$$T_o = \frac{T_1}{0.871} = \frac{300}{0.871} = 347.9 \text{ K}$$

Stagnation temperature T_o = 347.9 K

$$\left.\frac{P_1}{P_o}\right]_{M=0.86} = 0.617$$

$$P_o = \frac{P_1}{0.617} = \frac{1}{0.617} = 1.62$$

Stagnation pressure = 1.62 bar

Velocity pressure = 1.62 bar

Velocity of sound at stagnation temperature (C_o)

$$C_o = \sqrt{KRT_o}$$

$$= 20.05\sqrt{347.9}$$

C_o = 373.97 ms^{-1}

Q 5) Air at stagnation condition has a temperature of 700 K. Find the max. possible velocity. What would be the sonic velocity if flow velocity is 1/2 of the maximum velocity.

$$h + \frac{V^2}{2} = h_o$$

$$\frac{V^2}{2} = (h_o - h) = C_p (T_o - T)$$

V becomes maximum when T = 0

$$\frac{V^2_{max}}{2} = C_p T_o = 1005 \times 700 \cdot \frac{J}{Kg\,K} \cdot K$$

$$V_{max} = \sqrt{2 \times 1005 \times 700} = 1186.2 \text{ ms}^{-1}$$

Maximum possible flow velocity V_{max} = 1186.2 ms^{-1}

New flow velocity $V_1 = \frac{V_{max}}{2} = \frac{1186.2}{2} = 593.1 \text{ ms}^{-1}$

We have,

$$h_1 + \frac{V_1^2}{2} = h_o$$

$$C_p\ T_1 + \frac{(593.1)^2}{2} = C_p\ T_o$$

$$T_1 + \frac{(593.1)^2}{1005 \times 2} = T_o$$

$$T_1 + 700 - \frac{(593.1)^2}{2010}$$

T_1 = 524.99 K (525 K)

Sonic velocity

$$C_1 = \sqrt{KRT_1} = 20.05\sqrt{525}$$

C_1 = 459.4 ms^{-1}

Q 6) Air is discharged into the atmosphere from a big vessel through a nozzle. The atmospheric pressure and temperature are 1.1×10^5 N/m^2 and 40^{o}C and the jet velocity is 100 ms^{-1}. Find the chamber pressure and temperature.

Ans.

Mach number

$$M_1 = \frac{V_1}{C_1}$$

$$C_1 = \sqrt{KRT_1}$$

Fig.

$$= 20.05\sqrt{313} = 354.72 \text{ ms-1}$$

$$M_1 = \frac{V_1}{C_1}$$

$$= \frac{100}{354.72}$$

$M_1 = 0.28$

Stagnation properties (P_o, T_o)

From Gas tables we have,

$$\left.\frac{P_1}{P_o}\right]_{M=0.28} = 0.999$$

$$P_o = \frac{P_1}{0.999} = \frac{1.1\times10^5}{0.999} = 1.101 \times 10^5 \text{ N/m}^2$$

Chamber pressure $P_o = 1.101 \times 10^5$ N/m^2

$$\left.\frac{T_1}{P_o}\right]_{M=0.28} = 0.999$$

$$T_o = \frac{T_1}{0.999} = \frac{313}{0.999} = 313.33 \text{ K}$$

Chamber temperature $T_o = 313.33$ K

Q 7) Air stored in a big container t 400 K is allowed to expand adiabatically through a pipe. At a station (A) down stream in the pipe line the air attains a velocity of 250 m/s. Find velocity of sound in the container and at station A in the pipes.

Answer

Assumption - Friction losses are neglected

We have from SFEE,

$$h + \frac{V^2}{2} = h_o$$

Fig.

$$C_p T_1 + \frac{V^2}{2} = C_p T_o$$

$$C_p T_A + \frac{V^2{}_A}{2} = C_p T_o$$

$$T_A = T_o - \frac{V^2{}_A}{2C_p} = 400 - \frac{250^2}{2\times1005}$$

$T_A = 368.9$ K

Temperature at section (A) = 368.9 K

Velocity pof sound at station (A) (C_A)

$C_A = \sqrt{KRT_A} = 20.05\ \sqrt{368.9} = 385.1\ ms^{-1}$

Mach number $M_A = \dfrac{V_A}{C_A} = \dfrac{250}{385.1}$

$M_A = 0.65$

To find the temperature in the chamber

From Gas tables,

$$\left.\frac{T_A}{T_o}\right]_{M=0.68} = 0.915$$

$$T_o = \frac{T_A}{0.915} = \frac{368.9}{0.915}$$

Chamber temperature T_o = 403.17 K

Velocity of sound in the container (C_o)

$C_o = \sqrt{KRT_o} = 20.05\ \sqrt{403.17}$

$C_o = 402.6\ ms^{-1}$

Q30) Calculate the speed of sound in air at 400 K, if a bullet fired in this medium has a velocity of 650 ms^{-1}, find the mach angle of the wave produced by the bullet.

Ans:

$\sin\ a = \dfrac{C}{V}$

$C = \sqrt{KRT} = 20.05\sqrt{400}$ = 20.05 × 20 = 401 m/s

sin a = 401/650

$a = 38.09^o$

Q31) A supersonic fighter plane flics at an altide 3000 m. An observerver on the ground hears the sonic boom 7.5 seconds after the passing of the plane over his head. Estimate speed of plane in km/hr and mach number. The average temperature air is 11^{o}C.

Ans:

$$\cos a = \frac{7.5C}{3000}$$

$$C = \sqrt{KRT} = 20.05\sqrt{284} = 337.89 \text{ ms}^{-1}.$$

$$\cos a = \frac{7.5 \times 337.89}{3000}$$

$$a = 32.35^{o}$$

Speed of plane

$$\sin a = \frac{C}{V}$$

$$V = \frac{C}{\sin\alpha} \quad = \frac{337.89}{\sin 32.35^{o}}$$

$$V = 631.33 \text{ ms}^{-1}$$

$$V = 2272.78 \text{ Km/hr}$$

Mach number M = $\frac{V}{C} = \frac{631.33}{337.89} = 1.87$

Q. Show that the continuity equation for perfect gas can be represented in the form

$$\frac{dP}{P} + \frac{dA}{A} + \frac{dV}{V} - \frac{dT}{T} = 0$$

We have $\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$ .... (i)

For perfect gas equation of state,

$$P = \rho R T$$

$$\text{In } P = \text{In } \rho + \text{In } R + \text{In } T$$

$$\therefore \frac{dP}{P} = \frac{d\rho}{\rho} - \frac{dT}{T} \qquad \text{........ (ii)}$$

Using (ii) and (i) we have

$$\frac{dP}{P} + \frac{dA}{A} + \frac{dV}{V} - \frac{dT}{T} = 0$$

(ii) Law of conservation of Energy (First Law of Thermodynamics)

First law to a system:

Fig.

$$\delta q - \delta w = de]_{sys} \qquad \text{........ (1)}$$

On time basis

$$(1) \Rightarrow \delta\dot{q} - \delta\dot{w} = \frac{de}{dt}]_{sys} \qquad \text{........ (2)}$$

The system and C.V formulations are inter - related using Reynold's transport theorem:

$$\left.\frac{dN}{dt}\right]_{system} = \frac{\delta N}{\delta t} + \dot{N}_{out} - \dot{N}_{in} \qquad \text{........ (3)}$$

Where N is any property of system whose total change is required as system flows through CV.

Replacing N by e

$$\left.\frac{de}{dt}\right]_{sys} = \frac{\delta}{\delta t} e_{CV} + \dot{e}_{out} - \dot{e}_{in} \qquad \text{........(4)}$$

Figure represents flow through a control volume. The fluid property posses internal energy, K.E. and P.E. as it flows. The values of these energy changes continuously from inlet to exit. The specific values of these energy values at inlet and exit are shown on the sketch. The flow work done on to the C.V. is pv at inlet and flow work done by the CV at exit is (p + dp) (v + dv)

Assumptions

(i) 1 - D

(ii) Steady state

(iii) Compressible fluid

(iv) Higher order terms (HOT's) neglected

$$\dot{e}_{in} = u + \frac{v^2}{2} + gz$$

$$\dot{e}_{out} = (u + du) + \frac{(v + dv)^2}{2} + g(z + dz)$$

$\therefore\ \dot{e}_{out} - \dot{e}_{in}$ = du + vdv + gdz

Since steady state,

$$\frac{\delta}{\delta t} e_{CV} = 0$$

$\dot{\delta}_w$ represents the net flow work and the shaft work done by the CV

$$\dot{\delta}_w = \left[(p + dp)(v + dv) - pv\right] + \dot{\delta} w_{sh}$$

(2) combined with (4)

$$= \dot{\delta} q - \left\{\left[(p + dp)(v + dv) - pv\right] + \dot{\delta} w_{sh}\right\}$$

= 0 + du + vdv + gdz

$\dot{\delta} q - \dot{\delta} w_{sh}$ = pdv + vdp + du + vdu + gdz

Defining enthalpy,

h = u + pv

dh = du + pdv + vdp

$\therefore \dot{\delta} q - \dot{\delta} w_{sh}$ = dh + vdv + gdz ....(5)

Integrating (5)

$$\dot{q} - \dot{w}_{sh} = (h_2 - h_1) + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1)$$

This is called the steady flow energy equation (SFEE)

(iii) Law of conservation of momentum

(Newton's Second Law)

Law of conservation of momentum to a system

Fig.

$$\sum F_{xx} = \frac{d}{dt}(\text{momentum})\Big]_{sys} = \frac{d}{dt}(mv_{xx})\Big]_{sys}$$

$\frac{d}{dt}(mv_{xx})\Big]_{sys}$ is the total change of momentum and represents change of momentum with regard to time and change of momentum with respect to position.

The system and CV are interrelated through Reynold's transport theorem.

$$\frac{d}{dt}(mv_{xx})\Big]_{sys} = \frac{\partial}{\partial t}(mv_{xx})_{CV} + (\dot{m}v_{xx})_{out} - (\dot{m}v_{xx})_{in}$$

$$\therefore \sum F_{xx} = \frac{\partial}{\partial t}(mv_{xx})_{CV} + (\dot{m}v_{xx})_{out} - (\dot{m}v_{xx})_{in} \quad \text{........(i)}$$

Assumption

(i) 1 - D flow

(ii) Steady flow

(iii) Higher order terms (Hot's) are neglected

(iv) The fluid is invicid (M = 0)

(v) Compressible fluid

$\sum F_{xx}$

This represents the summation of all forces acting on CV and includes surface and body forces. Surface forces are of normal and tangential forms. Since the fluid is invicid shear force (tangential) = 0

Figure above shows a flow through CV. All fluid properties at inlet and exit are represented. Various forces acting are also represented in their respective direction at the appropriate locations.

$$\therefore \sum F_{xx} = F_{1xx} + F_{2xx} + F_{3xx} + F_{4xx}$$

$F_{1xx} = F_1 \,.\, pA$

$F_{2xx} = -F_2 = -(p + dp)(A + dA)$

$= -[pA + pdA + Adp]$ (neglecting HOT)

$F_{3xx} = F_3 \cos(90 - \alpha) = F_3 \sin \alpha$

$=$ (Average presure) $\times$(curved wall area) $\sin \alpha$

$$= \left(P + \frac{dp}{2}\right) A_w \sin\alpha$$

But $A_w \sin\alpha$ is the projection of the curved wall area on to a plane perpendicular to flow direction and equal to dA.

$$F_{3xx} = \left(P + \frac{dp}{2}\right) dA = pdA \quad \text{(neglecting HOT)}$$

$F_{4xx} = -$ F4 Cos $(90 - \theta)$

$= -$ F4 Sin θ

$= -$ mg Sin θ

$$= -\left[\left\{\left(A + \frac{dA}{2}\right) dx \left(\rho + \frac{d\rho}{2}\right)\right\} g \sin\theta\right]$$

m = mass of fluid within CV

= Volume of fluid $\times$ density

= (Avg. CSA) length $\times$ (Avg. density)

Also $dx \sin\theta = dz$

Fig.

$$F_{4xx} = -\left[\left\{\left(A+\frac{dA}{2}\right)dx\left(\rho+\frac{d\rho}{2}\right)\right\}g\,dz\right]$$

= - ρ Ag dz Neglecting HOT's ....

$\therefore \ \sum F_{xx}$ = pA - pA - pdA - Adp + pdA - ρ g Adz

$\sum F_{xx}$ = - Adp - ρ g Ag dz

RHS of (1)

$$= \frac{\partial}{\partial t}\left(mv_{xx}\right)_{CV} + \left(\dot{m}v_{xx}\right)_{out} - \left(\dot{m}v_{xx}\right)_{in}$$

Since steady flow

$$\frac{\partial}{\partial t}\left(mv_{xx}\right)_{CV} = 0$$

$$\left(\dot{m}v_{xx}\right)_{out} - \left(\dot{m}v_{xx}\right)_{in} = \dot{m}\left(v_{xx_{out}} - v_{xx_{in}}\right)$$

$$= \rho AV\left[\left(V+dV\right)-V\right]$$

$$= \rho AV\,dV$$

$$\therefore \text{RHS} = \rho AV\,dV$$

Equating LHS and RHS of (1)

-Adp - ρ Ag dz = ρ AV dV

Adp + ρ AVdV + ρ Agdz = 0

$\frac{d\rho}{\rho}$ + VdV + gdz = 0 Momentum equation - Euler's equation

<u>Integration of momentum equation</u>

Case 1: Incompressible fluid (density remains constant)

$$\therefore \ \int\frac{dp}{\rho} + \int VdV + \int gdz = \int 0$$

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant - Bernoullis equation}$$

Case 2 : Compressible fluid

$$\int \frac{dp}{\rho} + \int VdV + \int gdz = \int 0$$

$\frac{dp}{\rho}$ can be integrated by replacing ρ in terms of p, i.e., the relationship between ρ and p is to be established. The flow under consideration is a frictionless adiabatic (isentropic).

Hence $\frac{p}{\rho^k}$ = constant .... $\left(K = \frac{c_p}{c_v} = \gamma \right)$

$$\rho^k = \frac{p}{\text{Const}}$$

$$\rho = \left(\frac{p}{\text{Const}} \right)^{1/K}$$

$$\int \frac{dp}{\rho} = \int (P)^{-1/K} (\text{const})^{1/K} dp$$

$$= (\text{const})^{\frac{1}{K}} \left[\frac{P^{-1/K+1}}{-1/K+1} \right]$$

$$= \left(\frac{p}{\rho^K} \right)^{1/K} \frac{\left[p^{-1/K+1} \right]}{[-1/K+1]}$$

$$= \frac{\left(\frac{p}{\rho^K} \right)^{1/K} p^{-1/K+1}}{\frac{K-1}{K}}$$

$$= \frac{K}{K-1} \left(\frac{p}{\rho} \right)$$

$\therefore$ (3) takes the form

$$\frac{K}{K-1}\frac{p}{\rho}+\frac{V^2}{2}+gz = \text{constant}$$

Stagnation state

Fig.

the stagnation state is a reference state defined as that thermodynamic state which would exist if a fluid were brought to zero velocity and zero potential through an ideal path. The stagnation state must be reached, i) Without any energy exchange (Heat transfer = 0, work transfer = 0) ii) Without losses (frictionless)

Thus the stagnation process is isentropic.

Applying the SFEE for the process shown,

$$\dot{q}-\dot{w}_{sh} = \left(h_b - h_a\right)+\frac{\left(V_b^2 - V_a^2\right)}{2}+g\left(z_b - z_a\right)$$

$$h_b + \frac{V_b^2}{2} = h_a + \frac{V_a^2}{2} + gz_a \quad \left[\frac{V_b^2}{2} = 0\right]$$

Usually for gases potential changes are not accounted (being very negligible)

$$\therefore \; h_a + \frac{V_a^2}{2} = h_b$$

$$h_b = h_a + \frac{V_a^2}{2} \qquad \text{........ (1)}$$

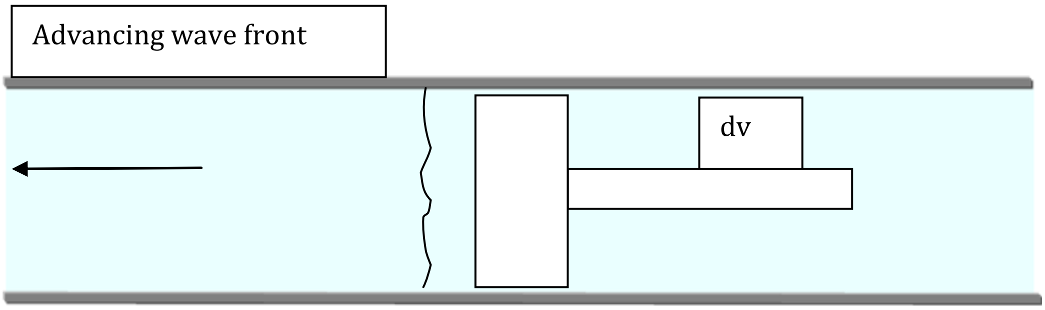
Here state (b) represents the stagnantion state corresponding to state (a). All properties at the stagnation state are identified by the subscripts ‘0’. Therefore generalizing (1)

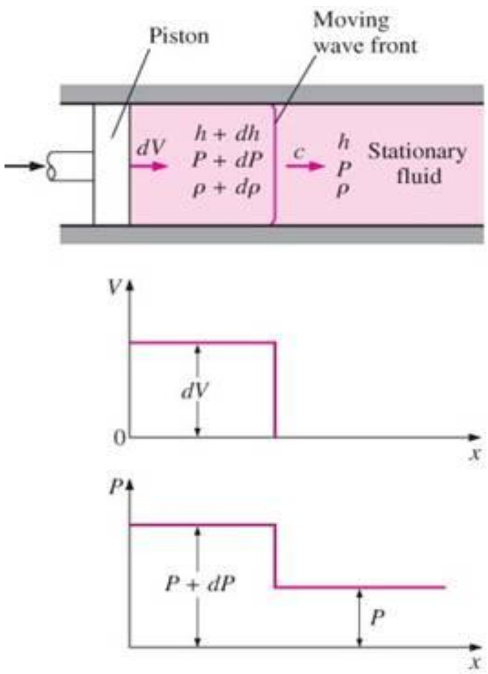
$$h_o = h + \frac{V^2}{2}$$

When the frame of reference is changed, the stagnation condition will also change, although the static condition remains the same.

Velocity of sound

The physiological effects of speeds and hearing are related to the transmission and detection of tiny pressure disturbances, and the speed at which these disturbances travel is known as the speed of sound. This wave velocity depends on the specific properties of the transmitting fluids and the magnitude of the pressure disturbances. Waves come under various strength, which are measured by the amplitude of the disturbances. The stronger the wave is the faster it moves.In order to relate these effects in a qualitative way consider the piston tube arrangement shown.





If the piston is made to move very slowly with velocity dv as shown, the fluid is compressed in front of the piston and the pressure and density increase slightluy to values (p + dp) and $(\rho + d\rho)$. Experience shows that this change (pressure wave front) will move along the tube at some steady speed c. Behind this advancing front, the fluid properties remain constant at their increased values, as long as piston continues to move steadily. Because of the continuity condition, the fluid behind the advancing front, moves at the speed of the piston (dv) while the fluid in front of the advancing wave front is still stationary.

The flow situation is unsteady in a fixed reference frame, and calculations become complicated. Hence we consider a moving reference frame, so as to make steady state.

Under the restrictions of

(i) Small pressure difference

(ii) Frictionless movement

(iii) No heat flow

the process can be considered isentropic

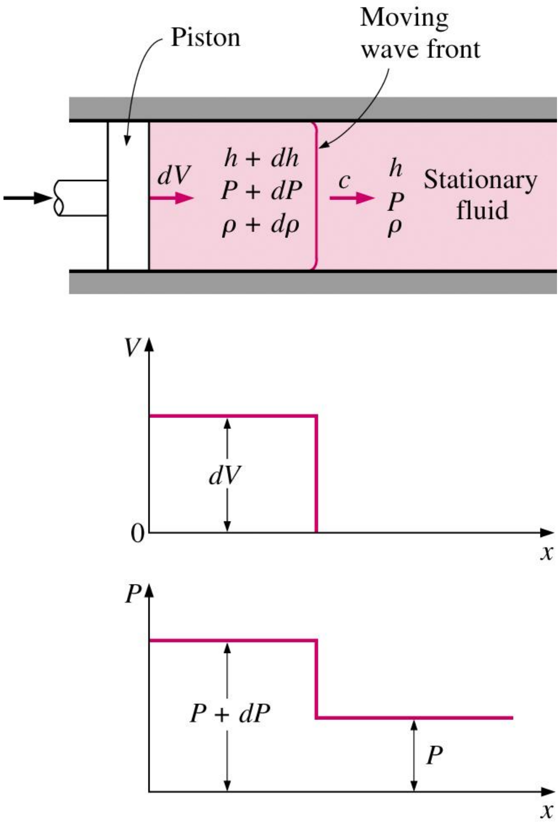
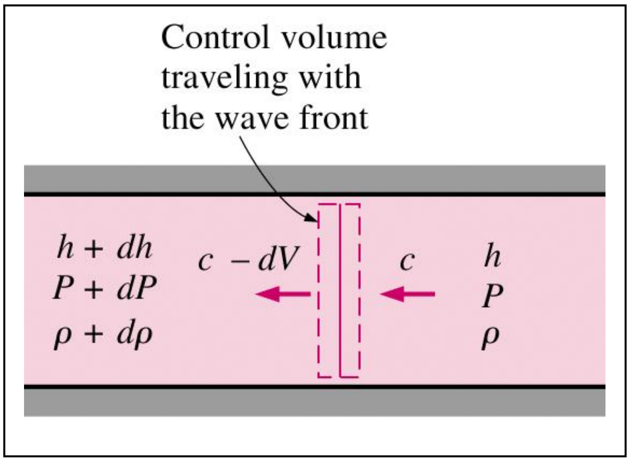
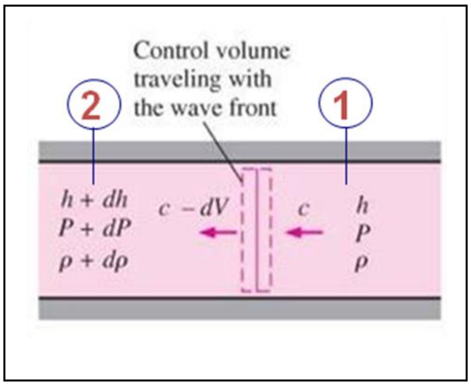


Fig (a) observer on moving reference frame.

Fig (b) observer on fixed frame.





Basic Law's of flow situations

(i) Continuity :

$$\dot{m}_{in} = \dot{m}_{out}$$

$$(\rho AV)_{in} = (\rho AV)_{out}$$

$$\rho AC = (\rho + dp)A(C - dv)$$

$$= \rho AC - \rho Adv + CAd\rho$$

$$\rho Adv = CAd\rho$$

$$dv = \frac{Cd\rho}{\rho} \qquad \text{........ (1)}$$

(ii) Momentum

$$\sum F_{xx} = \dot{m}\left[(V_{xx})_{out} - (V_{xx})_{in}\right]$$

$$PA - (p + dp)\,A = \rho AC\,(C - dv - c)$$

$$- Adp = \rho AC\,(-dv)$$

$$dv = \frac{dp}{\rho c} \qquad \text{........ (2)}$$

Equating (1) and (2)

$$\frac{dp}{\rho c} = \frac{Cd\rho}{\rho}$$

$$c^2 = \frac{dp}{d\rho}$$

But $\frac{dp}{d\rho}$ is not unique, it depends on the process. The process considered here is being isentropic, $\frac{dp}{d\rho}$ represents that for an isentropic process for which $\frac{p}{\rho^k}$ = constant.

$$\because\ P = \rho^k$$

$$\therefore C^2 = \frac{dp}{d\rho} = \left.\frac{\partial P}{\partial \rho}\right]_s$$

$$\frac{dp}{d\rho} = K\rho^{K-1} \text{ x constant}$$

$$= K\rho^{K-1}\frac{P}{\rho^k}$$

$$\left.\frac{\partial P}{\partial \rho}\right]_s = K\frac{p}{\rho}$$

$$\frac{dP}{d\rho} = K\frac{P}{\rho}$$

Considering the medium as a perfect gas with equation of state

$P = \rho RT$

$$C^2 = \frac{dp}{d\rho} = \left.\frac{\partial P}{\partial \rho}\right]_s = K\frac{p}{\rho} = KRT$$

$$C = \left.\frac{\partial P}{\partial \rho}\right]_s = \sqrt{K\frac{P}{\rho}} = \sqrt{KRT}$$

$C^2 = KRT$

$C2 = K\frac{\bar{R}}{M.W}T$ (M.W = molecular weight)

mass balance: $\overset{o}{m}_1 = \overset{o}{m}_2$

$$\rho Ac = (\rho + d\rho)A(c - dV)$$

$$cd\rho - \rho dV = 0 \qquad \text{.............(a)}$$

Energy Balance: SSSF

$$h + \frac{c^2}{2} = (h + dh) + \frac{(c - dV)^2}{2}$$

$$dh = \frac{dP}{\rho} \qquad \text{...............(b)}$$

Thermodynamic Relation: $Tds = dh - vdP$

~Isentropic$(ds = 0)$: $\quad dh = \dfrac{dP}{\rho} \qquad$ ....(c)

(a), (b), and (c) $\rightarrow$

$$c^2 = \frac{dP}{d\rho}; \qquad \text{at } s = \text{ const}$$

Isentropic of ideal gas: $Pv^k = const.$

or...... $\dfrac{dP}{P} - k\dfrac{d\rho}{\rho} = 0; \quad \rightarrow \quad \dfrac{dP}{d\rho} = k\dfrac{P}{\rho}$

eq 17.9.. $\quad c^2 = k\dfrac{P}{\rho} = kPv = kRT$

$$c = \sqrt{kRT}$$

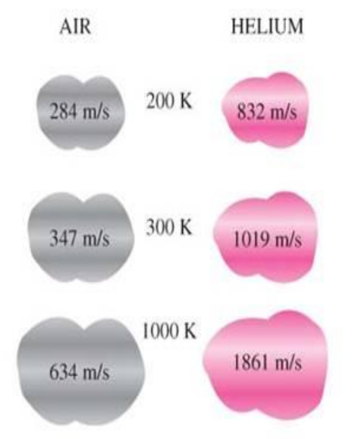


FIGURE 17–9

The speed of sound changes with temperature and varies with the fluid.

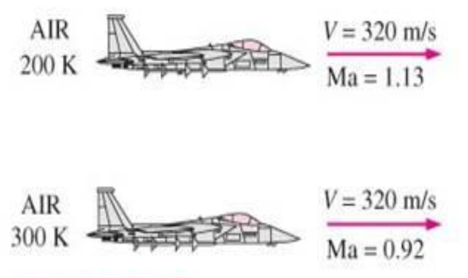
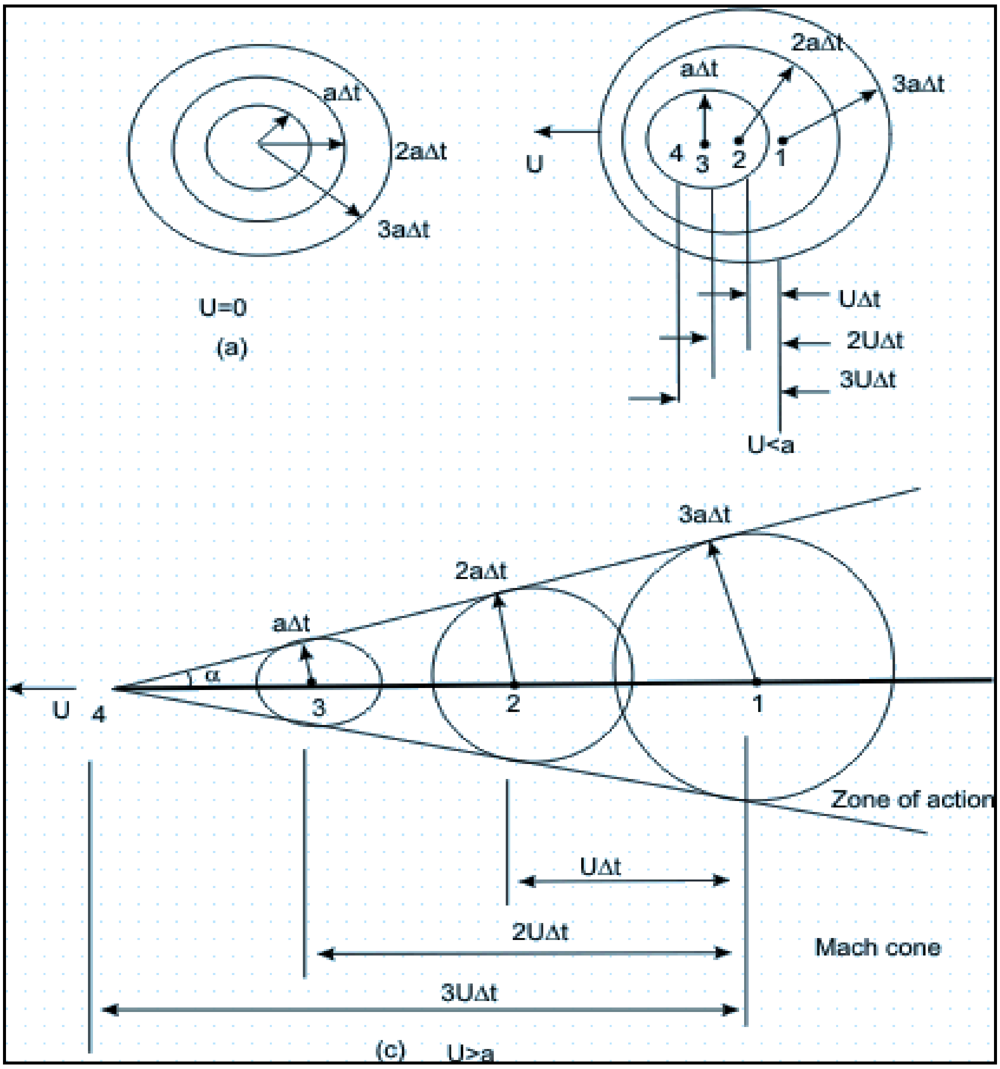


FIGURE 17–10

The Mach number can be different at different temperatures even if the velocity is the same.

Pressure Field Due to a Moving Source

- Consider a point source emanating infinitesimal pressure disturbances in a still fluid, in which the speed of sound is "a". If the point disturbance, is stationary then the wave fronts are concentric spheres. As shown in Fig. (a), wave fronts are present at intervals of Δt .
- Now suppose that source moves to the left at speed U < a**.** Figure (b) shows four locations of the source, 1 to 4, at equal intervals of time Δt , with point 4 being the current location of the source.
- At point 1, the source emanated a wave which has spherically expanded to a radius $3a\Delta t$ in an interval of time $3\Delta t$. During this time the source has moved to the location 4 at a distance of $3u\Delta t$ from point 1. The figure also shows the locations of the wave fronts emitted while the source was at points 2 and 3, respectively.
- When the source speed is supersonic U > a (Fig. (c)), the point source is ahead of the disturbance and an observer in the downstream location is unaware of the approaching source. The disturbance emitted at different



Wave fronts emitted from a point source in a still fluid when the source speed is

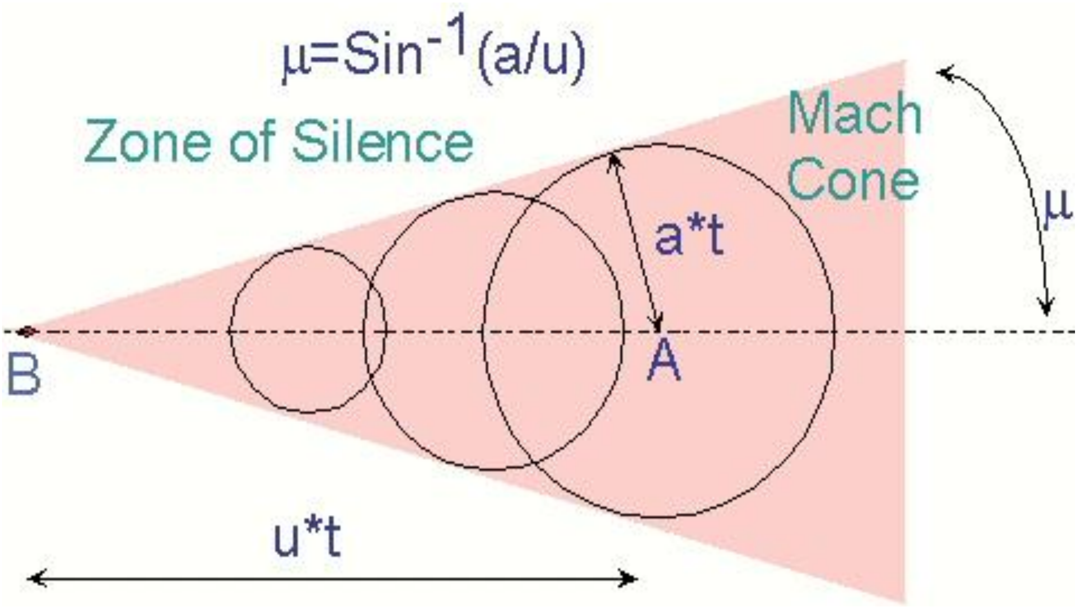
(a) U = 0 (still Source) (b) U < a (Subsonic) (c) U > a (Supersonic)

Points of time are enveloped by an imaginary conical surface known as "Mach Cone". The half angle of the cone $\propto$, is known as Mach angle and given by

$$\sin\alpha = \frac{a\Delta t}{U\Delta t} = \frac{1}{Ma}$$
$$\alpha = \sin^{-1}(1/Ma)$$

- **Since the disturbances are confined to the cone,** the area within the cone is known as zone of action **and the** area outside the cone is zone of silence**.**

An observer does not feel the effects of the moving source till the Mach Cone covers his position



Mach angle

As the aircraft flies faster than the speed of sound (M > 1), the shock wave forms either a wedge or a cone. Mach waves are very weak shock waves when the disturbance is very small. If this airplane travels at the speed of U [m/s], the aircraft will move U*t [m] (shown in the figure above as the distance AB), while the pressure wave which is formed at the initial position, will travel the distance of a*t [m] (shown as AC). Hence the half angle of the Mach wave," ", should satisfy the following equation.

= sin-1((a*t)/(U*t)

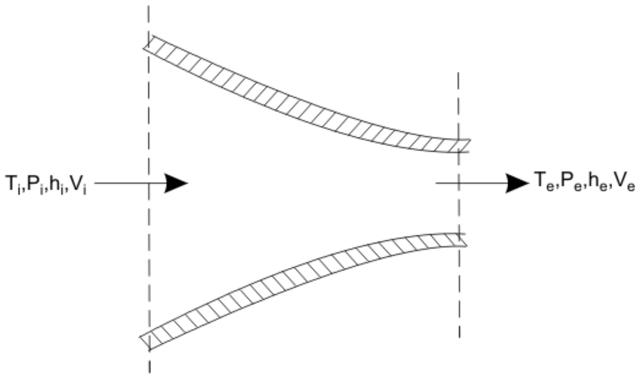
•

Flow through Nozzles

A *nozzle* is a duct that increases the velocity of the flowing fluid at the expense of pressure drop. A duct which decreases the velocity of a fluid and causes a corresponding increase in pressure is a *diffuser*. The same duct may be either a nozzle or a diffuser depending upon the end conditions across it. If the cross-section of a duct decreases gradually from inlet to exit, the duct is said to be convergent. Conversely if the cross section increases gradually from the inlet to exit, the duct is said to be divergent. If the cross-section initially decreases and then increases, the duct is called a convergent-divergent nozzle. The minimum cross-section of such ducts is known as throat. A fluid is said to be *compressible* if its density changes with the change in pressure brought about by the flow. If the density does not changes or changes very little, the fluid is said to be incompressible. Usually the gases and vapors are compressible, whereas liquids are *incompressible.*

Nozzle:

A nozzle is primarily used to increase the flow velocity.



The first law reduces to

$$h_e + \frac{V_e^2}{2} = h_i + \frac{V_i^2}{2}$$

Or

$$V_e^2 - V_i^2 = 2(h_i - h_e)$$

If the inlet velocity is negligible $V_i \approx 0$ *and then*

$$V_e \sqrt{2\left(h_i - h_e\right)}$$

$$V_e \sqrt{2\left(h_i - h_e\right) + V_i^2}$$

The velocity is increased at the cost of drop in enthalpy. If an ideal gas is flowing through the nozzle, the exit velocity V_e can be expressed in terms of inlet and outlet pressure and temperatures by making use of the relations

$$P\upsilon = RT$$

$$dh = c_p dT$$

.

Therefore,

$$V_e^2 = 2c_p\left(T_i - T_e\right) = 2c_p T_i\left(1 - \frac{T_e}{T_i}\right)$$

From the relations governing adiabatic expansion,

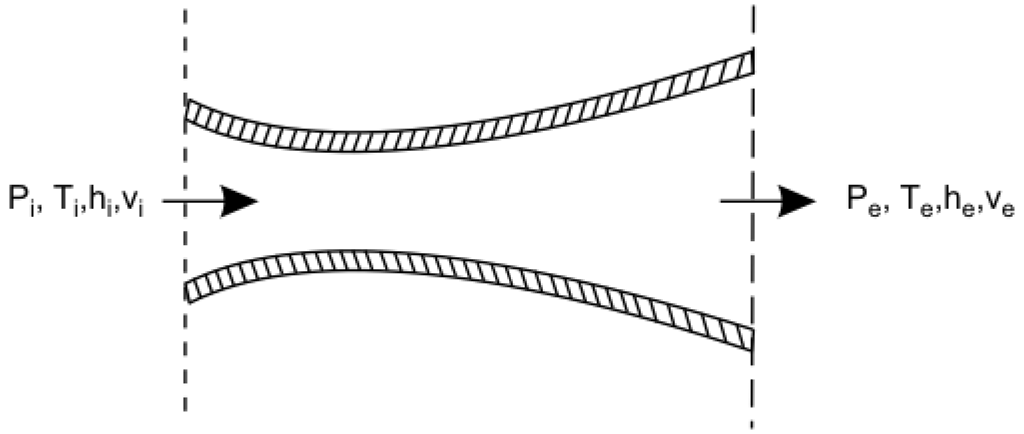
$$\frac{T_e}{T_i} = \left(\frac{P_e}{P_i}\right)^{\frac{\gamma-1}{\gamma}}$$

We get,

$$V_e = \sqrt{2C_p T_i\left[1 - \left(\frac{P_e}{P_i}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Diffuser:

A diffuser can be thought of as a nozzle in which the direction of flow is reversed.



Figure

For an adiabatic diffuser, Q and W_{sh} are zero and the first law reduces to

$$h_e + \frac{V_e^2}{2} = h_i + \frac{V_i^2}{2}$$

$$V_e^2 - V_i^2 = 2(h_i - h_e)$$

The diffuser discharges fluid with higher enthalpy. The velocity of the fluid is reduced.

Isentropic Flow

• Adiabatic: no heat is added or taken away ($\delta q=0$)

• Reversible: no frictional or other dissipative effects

• Isentropic: BOTH! (Adiabatic + Reversible)

Example of a reversible process:

– Slow compression of air in a balloon does work on the air inside the balloon, and takes away energy from the surroundings - When the balloon is allowed to expand, the air inside and the surrounding air are both restored to original conditions

• Example of an irreversible process:– Heat flows from hot to cold, never in the opposite direction; Most conductive and viscous processes are irreversible
Stagnation point is thus when fluid is brought to stagnant state (eg, reservoir)

Stagnation properties can be obtained at any point in a flow field if the fluid at that point were decelerated from local conditions to zero velocity following an isentropic (frictionless, adiabatic) process

Pressure: p_0

Temperature: T_0

Density: ρ_0

If a fluid were brought to a complete stop ($C_2 = 0$)

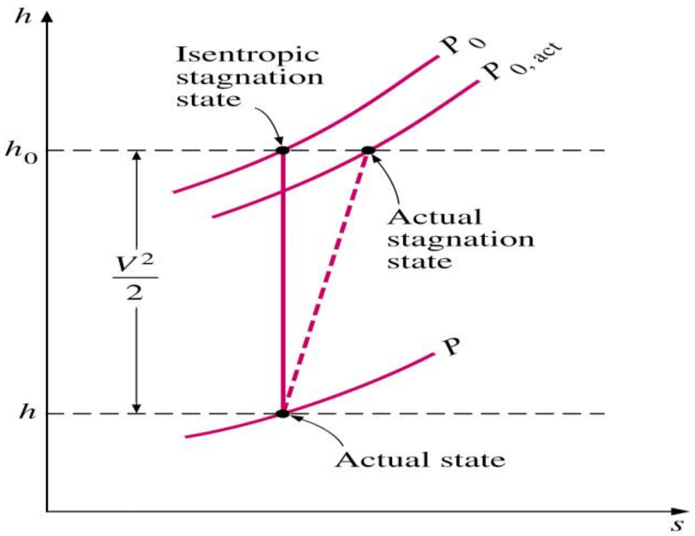
Therefore, h_0 represents the enthalpy of a fluid when it is brought to rest adiabatically.

During a stagnation process, kinetic energy is converted to enthalpy.

Properties at this point are called **stagnation properties** (which are identified by subscript 0)

If the process is also reversible, the stagnation state is called the **isentropic stagnation state.**

Stagnation enthalpy is the same for isentropic and actual stagnation states



Actual stagnation pressure $P_{0,act}$ is lower than P_0 due to increase in entropy s as a result of fluid friction.Nonetheless, stagnation processes are often approximated to be isentropic, and isentropic properties are referred to as stagnation properties

Types of nozzles and diffusers

Three types of nozzles:

1. Converging,.Diverging,. Converging – diverging

1. Converging nozzle, : In convergent nozzle,the cross sectional area decreases fron inlet section to the out let section.

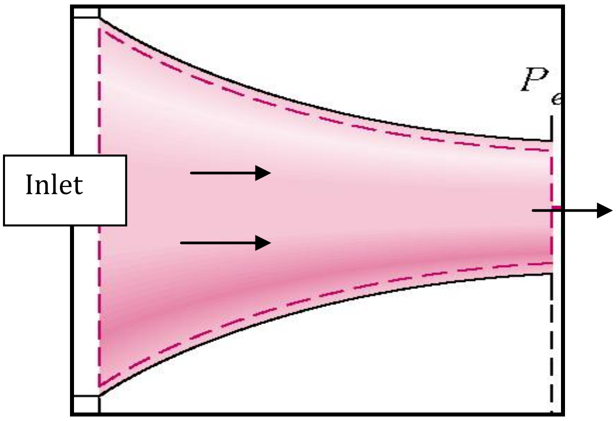


Figure 1 converging nozzle

Diverging nozzle,. In convergent nozzle, the cross sectional area increases fron inlet section to the out let section.

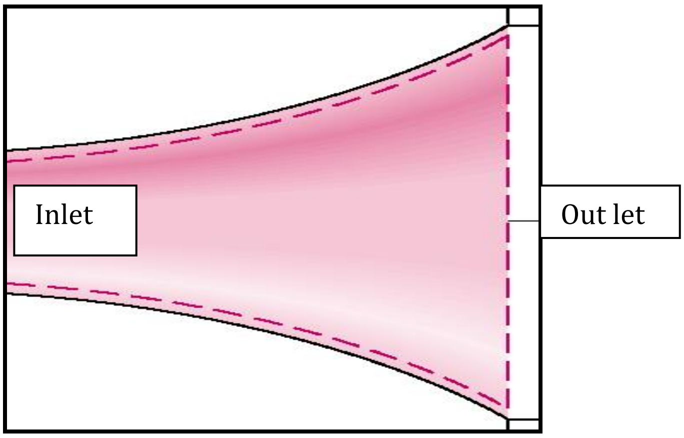
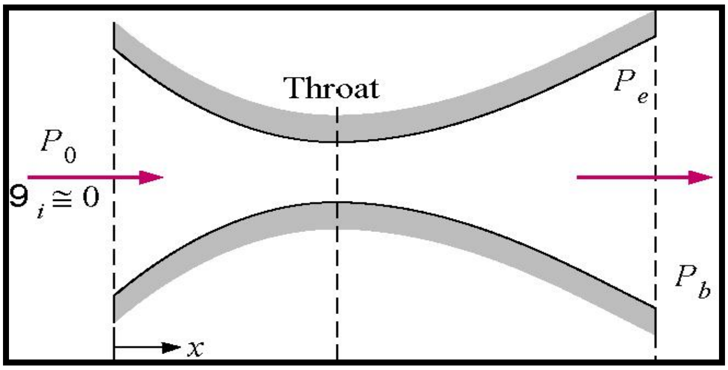


Figure 2Divergent nozzle

Converging – diverging

In convergent divergent nozzle the cross sectional area first decreases from inlet section to the throat and the increases from its throat to outlet section



Effect of Area Variation on Flow Properties in Isentropic Flow

from stagnation enthalpy equation, we know that

$$h_0 = h + \frac{C^2}{2}$$

$$h+\frac{c^2}{2}=h_0=C$$

differentiating this equation

$$dh+\frac{2cdc}{2}=0$$

$$dh+cdc=0$$

we know that for isentropic flow

$$dh=\frac{dp}{\rho}$$

$$\frac{dp}{\rho}+cdc=0$$

$$\frac{dp}{\rho}=-c\ dc$$

$$dp=-\rho c\ dc$$

mass flowrate, $m=\rho Ac=C$

$$\rho Ac=C$$

taking log *on both sides*

$$In\,\rho+In\,A+In\,c=0$$

Differentiating

$$\frac{d\rho}{\rho}+\frac{dA}{A}+\frac{dc}{c}=0$$

$$\frac{dc}{c}=-\left[\frac{d\rho}{\rho}+\frac{dA}{A}\right]$$

$$dc=-c\left[\frac{d\rho}{\rho}+\frac{dA}{A}\right]$$

Substituing dc value in Equation

$$dp = -\rho c\ dc$$

$$dp = -\rho c\left[-c\left[\frac{d\rho}{\rho} + \frac{dA}{A}\right]\right]$$

$$dp = \rho \mathrm{c}^2\left[\frac{\mathrm{d}\rho}{\rho} + \frac{dA}{A}\right]$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} = \frac{dp}{\rho c^2}$$

$$\frac{dA}{A} = \frac{dp}{\rho c^2} - \frac{d\rho}{\rho}$$

$$= \frac{dp}{\rho c^2}\left[1 - \frac{\rho c^2}{dp} * \frac{\mathrm{d}\rho}{\rho}\right]$$

$$\frac{dA}{A} = \frac{dp}{\rho c^2}\left[1 - c^2 * \frac{d\rho}{dP}\right]$$

For isentropic process

$$\frac{dp}{d\rho} = a^2$$

$$\frac{dA}{A} = \frac{dp}{\rho c^2}\left[1 - \frac{c^2}{a^2}\right]$$

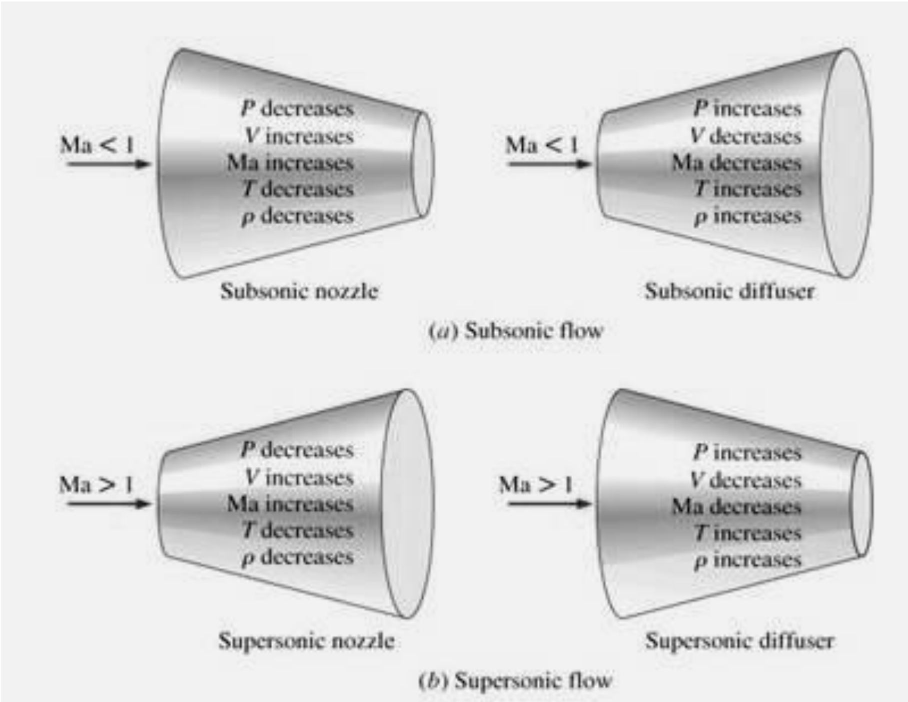
Mach number, $M = \dfrac{c}{a}$

$$\frac{dA}{A} = \frac{dp}{\rho c^2}\left[1 - M^2\right]$$

This equation is considered for increasing and decreasing area passage for various values of Mach number.

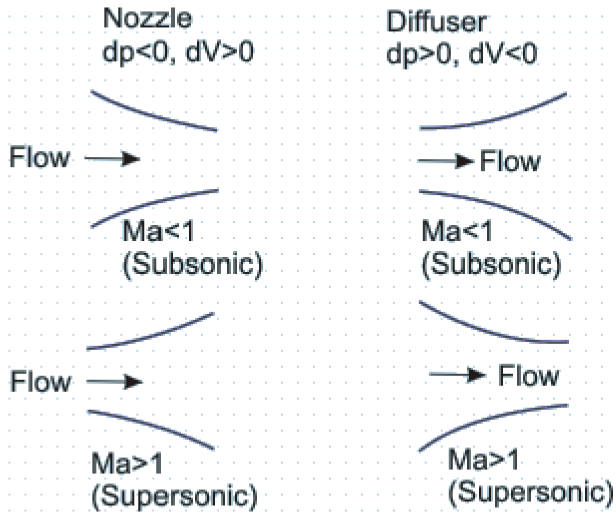
Nozzle/Diffuser

- Subsonic flow (Ma<1), dA/dV < 0
- Sonic flow (Ma=1), dA/dV = 0
- Supersonic flow (Ma>1), dA/dV > 0



At subsonic speeds **(**Ma<1**)** a decrease in area increases the speed of flow. A subsonic nozzle should have a convergent profile and a subsonic diffuser should possess a divergent profile. The flow behavior in the regime of Ma<1 is therefore qualitatively the same as in incompressible flows.

1. In supersonic flows **(**Ma>1**)** the effect of area changes are different. According to Eq. (40.11), a supersonic nozzle must be built with an increasing area in the flow direction. A supersonic diffuser must be a converging channel. Divergent nozzles are used to produce supersonic flow in missiles and launch vehicles.

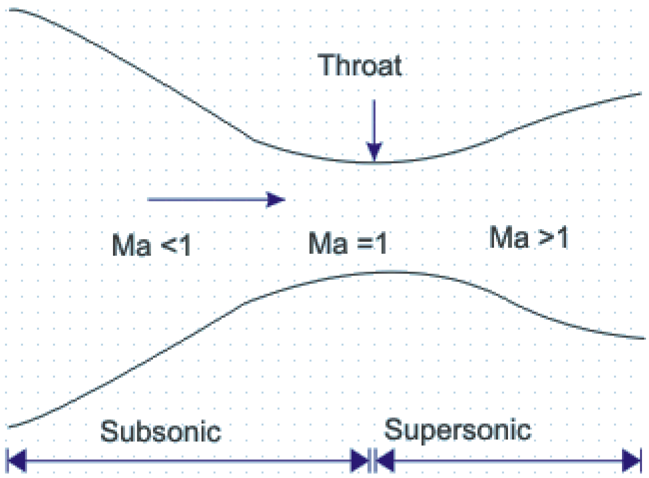


Shapes of nozzles and diffusers in subsonic and supersonic regimes

Convergent - Divergent Nozzle

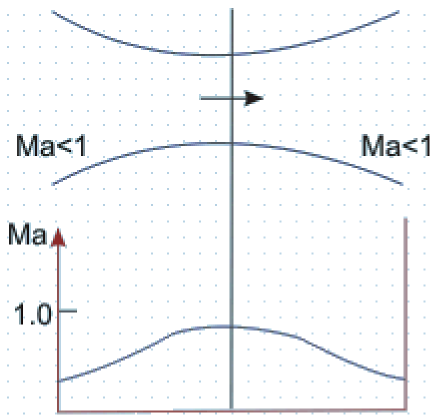
Suppose a nozzle is used to obtain a supersonic stream starting from low speeds at the inlet (Then the Mach number should increase from Ma=0 near the inlet to Ma>1 at the exit. It is clear that the nozzle must converge in the subsonic portion and diverge in the supersonic portion. Such a nozzle is called a **convergent-divergent nozzle**. A convergent-divergent nozzle is also called a **de laval nozzle**, after Carl G.P. de Laval who first used such a configuration in his steam turbines in late nineteenth century.

it is clear that the Mach number must be unity at the throat, where the area is neither increasing nor decreasing. This is consistent with which shows that dV can be nonzero at the throat only if Ma =1. It also follows that the **sonic velocity can be achieved only at the throat of a nozzle or a diffuser**.



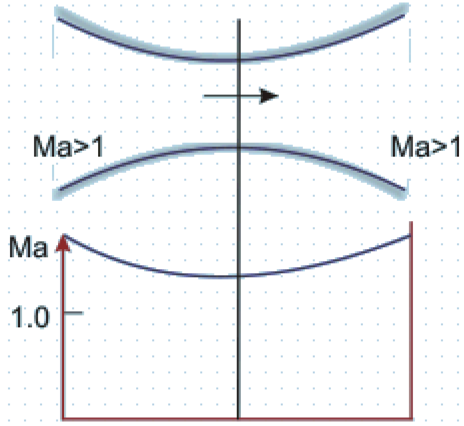
A Convergent-Divergent Nozzle

The condition, however, does not restrict that Ma must necessarily be unity at the throat. According to situation is possible where Ma ≠ 1 at the throat if **dV = 0** there. For an example, the flow in a convergent-divergent duct may be subsonic everywhere with Ma increasing in the convergent portion and decreasing in the divergent portion with Ma ≠ 1 at the throat



Convergent-Divergent duct with Ma ≠ 1 at throat

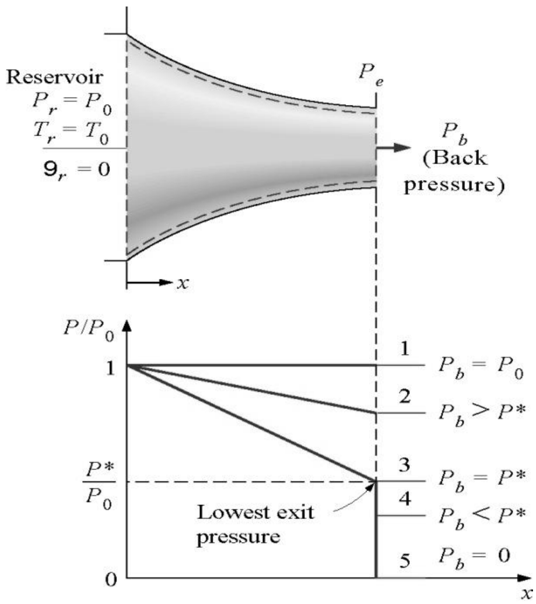
The first part of the duct is acting as a nozzle, whereas the second part is acting as a diffuser. Alternatively, we may have a convergent divergent duct in which the flow is supersonic everywhere with Ma decreasing in the convergent part and increasing in the divergent part and again Ma ≠ 1 at the throat

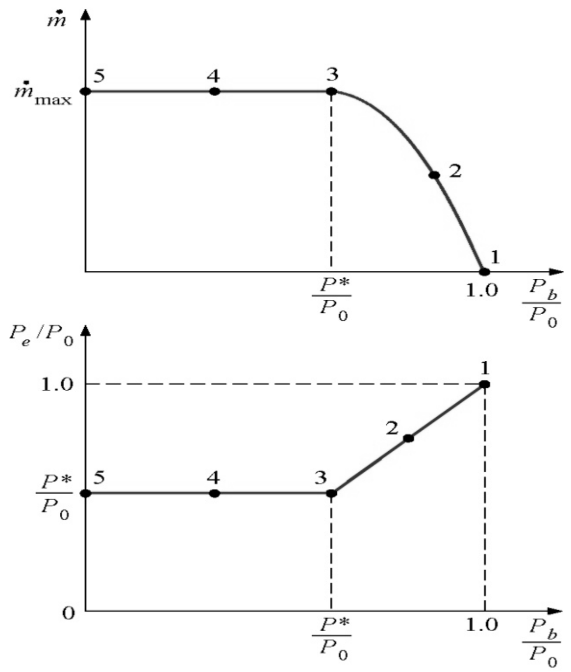


Convergent-Divergent duct with Ma ≠ 1 **at throat**

Effect of Back Pressure on Flow through a Converging Nozzle

Consider the converging nozzle shown below. The flow is supplied by a reservoir at pressure P_0 and temperature T_0. The reservoir is large enough that the velocity in the reservoir is zero. Let's plot the ratio P/P_o along the length of the nozzle, the mass flow rate through the nozzle, and the exit plane pressure P_e as the back pressure P_b is varied. Let's consider isentropic flow so that P_o is constant throughout the nozzle.





1. $P_b = P_o$, $P_b / P_o = 1$. No flow occurs. $P_e = P_b$, M_e=0.

2. $P_b > P^*$ or $P^*/P_o < P_b / P_o < 1$. Flow begins to increase as the back pressure is lowered. $P_e = P_b$, $M_e < 1$.

3. $P_b = P^*$ or $P^*/P_o = P_b / P_o < 1$. Flow increases to the choked flow limit as the back pressure is lowered to the critical pressure. $P_e = P_b$, M_e=1.

4. $P_b < P^*$ or $P_b / P_o < P^*/P_o < 1$. Flow is still choked and does not increase as the back pressure is lowered below the critical pressure, pressure drop from P_e to P_b occurs outside the nozzle. $P_e = P^*$, M_e=1.

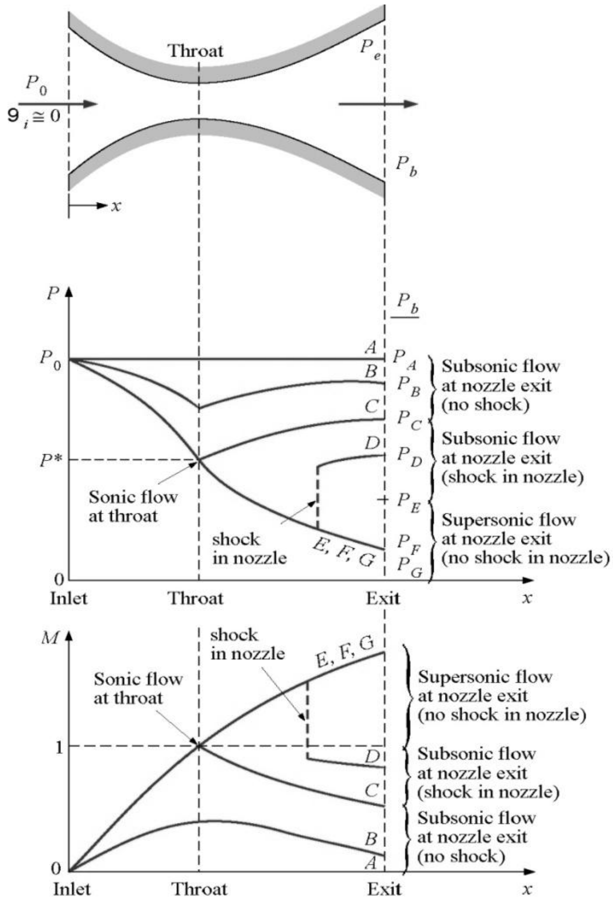
5. $P_b = 0$. Results are the same as for item 4.Consider the converging-diverging nozzle shown below.

Let's plot the ratio P/P_o and the Mach number along the length of the nozzle as the back pressure P_b is varied. Let's consider isentropic flow so that P_o is constant throughout the nozzle.

- $P_A = P_o$, or $P_A/P_o = 1$. No flow occurs. $P_e = P_b$, $M_e = 0$.

  $P_o > P_B > P_C > P^*$ or $P^*/P_o < P_C/P_o < P_B/P_o < 1$. Flow begins to increase as the back pressure is lowered. The velocity increases in the converging section but $M < 1$ at the throat; thus, the diverging section acts as a diffuser with the velocity decreasing and pressure increasing. The flow remains subsonic through the nozzle. $P_e = P_b$ and $M_e < 1$.

$P_b = P_C = P^*$ or $P^*/P_o = P_b/P_o = P_C/P_o$ and P_b is adjusted so that M=1 at the throat. Flow increases to its maximum value at choked conditions; velocity increases to the speed of sound at the throat, but the converging section acts



- as a diffuser with velocity decreasing and pressure increasing. $P_e = P_b$, $M_e < 1$.

- $P_C > P_b > P_E$ or $P_E/P_o < P_b/P_o < P_C/P_o < 1$. The fluid that achieved sonic velocity at the throat continues to accelerate to supersonic velocities in the diverging section as the pressure drops. This acceleration comes to a sudden stop, however, as a normal shock develops at a section between the throat and the exit plane. The flow across the shock is highly irreversible. The normal shock moves downstream away from the throat as P_b is decreased and approaches the nozzle exit plane as P_b approaches P_E. When $P_b = P_E$, the normal shock forms at the exit plane of the nozzle. The flow is supersonic through the entire diverging section in this case, and it can be approximated as isentropic. However, the fluid velocity drops to subsonic levels just before leaving the nozzle as it crosses the normal shock.

- $P_E > P_b > 0$ or $0 < P_b/P_o < P_E/P_o < 1$. The flow in the diverging section is supersonic, and the fluids expand to P_F at the nozzle exit with no normal shock forming within the nozzle. Thus the flow through the nozzle can be approximated as isentropic. When $P_b = P_F$, no shocks occur within or outside the nozzle. When $P_b < P_F$, irreversible mixing and expansion waves occur downstream of the exit plane or the nozzle. When $P_b > P_F$, however, the pressure of the fluid increases from P_F to P_b irreversibly in the wake or the nozzle exit, creating what are called oblique shocks.

EXPLAIN UNDER EXPANDED AND OVEREXPANDED NOZZLE?

Normally the cross-section al area decreases in the flow direction. However the maximum velocity to which the fluid can be accelerated is sonic velocity (M=1). Accelerating a fluid to supersonic velocities can be achieved only by attaching diverging flow passage to the exit of subsonic nozzle at the throat (M>1),resulting in a configuration known as convergent-divergent nozzle.

Consider a convergent –divergent nozzle which is fitted to a tank of infinitely large size, hence the inlet of the nozzle, pressure is equal to Po (stagnation pressure). A fluid enters with a low velocity at stagnation pressure Po. When (Pb=Po), There will be no flow through the nozzle, since the pressure difference is zero.

When the back pressure of the nozzle 'Pb' below the design pressure 'Pd', the nozzle is said to be under-expanding. In under-expansion the fluid enters up to the design pressure 'Pd' in the nozzle and expands violently and irreversibly down to the back pressure 'Pb' after leaving the nozzle and outside it. The jet will exit in a diverging stream.

Over-expanded flow

Under-expanded flow

When the back pressure of the nozzle is above the design value, the nozzle is said to be over-expanding. In overexpansion in a convergent nozzle the exit pressure is greater than the critical pressure and the effect is to reduce the mass flow rate through the nozzle. In over-expansion in a convergent nozzle the exit pressure is greater than critical pressure and the effect is to reduce the mass flow rate through the nozzle. In over-expansion in a convergent-divergent nozzle, there is always an expansion followed by compression. The flow will exit the jet in a converging stream. While analyzing the flow, the two types of nozzle should be considered separately.

Nozzle and diffuser efficiencies

In ideal case, flow through nozzle and diffuser is isentropic. But in actual case, friction exists and affects in following ways:

i) Reduces the enthalpy drop reduces the final velocity of steam

iii) Increases the final dryness fraction

iv) Increases specific volume of the fluid

v) Decreases the mass flow rate

Nozzle performance

The isentropic operating conditions are very easy to determine. Frictional losses in the nozzle can be accounted by several methods.

(1) Direct information on the entropy change could be given although this is usually not available.

(2) Some times equivalents information is provided in the form of stagnation pressure ratio. Normally nozzle performance is indicated by efficiency parameter defined as

$\eta_N = \dfrac{\text{actual change in KE}}{\text{total change in KE}}$ for the nozzle operating to same back pressure for both expansion

$$\text{i.e., } \eta_N = \left.\frac{(\Delta\ KE)_{actual}}{(\Delta\ KE)_{ideal}}\right]_{Pb}$$

From SFEE

$$h + \frac{V^2}{2} = h_o$$

$$\Delta\ KE\big]_{actual} = h_o - h_a$$

$$\Delta\ KE\big]_{ideal} = h_o - h_i$$

$$\text{i.e., } \eta_N = \left.\frac{h_o - h_a}{h_o - h_i}\right]_{Pb}$$

$$\eta_N = \left.\frac{T_o - T_a}{T_o - T_i}\right]_{Pb}$$

Velocity Coefficient (C_V)

$C_V = \dfrac{\text{Actual outlet velocity}}{\text{Ideal outlet velocity}} =$ the nozzle operating to the same back pressure

$$C_V = \left.\frac{V_o}{V_i}\right]_{Pb}$$

$$C_V^2 = \frac{V_a^2}{V_i^2} = \frac{V_a^2/2}{V_i^2/2} = \eta_N$$

$C_V = \sqrt{\eta_N}$ for initial state at stagnation point

Discharge co-efficient (C_d)

$C_d = \dfrac{\text{Actual mass flow rate}}{\text{Total mass flow rate}}$ nozzle operating for same back pressure

$$= \left.\frac{\dot{m}_a}{\dot{m}_i}\right]_{Pb}$$

$$= \left.\frac{(\rho AV)_a}{(\rho AV)_i}\right]_{Pb}$$

$$C_d = \left.\frac{\rho_a V_a}{\rho_i V_i}\right]_{Pb}$$

Q. Show that $C_d\ C_v = \dfrac{M_a^2}{M_i^2}$

$$C_d = \left.\frac{\rho_a V_a}{\rho_i V_i}\right]_{Pb} \qquad C_V = \frac{V_a}{V_i}$$

$$C_d\ C_v = \frac{\rho_a V_a V_a}{\rho_i V_i V_i} = \frac{\rho_a V_{a\,a}^2}{\rho_i V_i^2}$$

$P = \rho RT$ for an ideal gas

$$\rho = \frac{p'}{RT} \text{ .... } P_i = P_a = P_b$$

$$\frac{\rho_a}{\rho_i} = \frac{P_a / RT_a}{P_i / RT_i}$$

$$\frac{\rho_a}{\rho_i} = \frac{1/T_a}{1/T_i}$$

$$C_d\ C_v = \frac{\frac{1}{T} V_a^2}{\frac{1}{T_i} V_i^2} = \frac{\frac{V_a^2}{KR\,T_a}}{\frac{V_i^2}{KR\,T_i}}$$

$$= \frac{\left(\frac{V_a^2}{C_a^2}\right)}{\left(\frac{V_i^2}{C_i^2}\right)} \qquad \text{But } KRT = C^2$$

$\therefore\ C_d\, C_v = \dfrac{M_a^2}{M_i^2}$

Note : If the nozzle inlet is not stagnant.

$$\eta_N = \frac{\frac{V_{2a}^2}{2} - \frac{V_i^2}{2}}{\frac{V_{2i}^2}{2} - \frac{V_i^2}{2}}$$

$$= \frac{h_1 - h_{2a}}{h_1 - h_{2i}}$$

$$\eta_N = \frac{T_1 - T_{2a}}{T_1 - T_{2i}}$$

Note: (i) Non Steep Flow

$$M_w = \frac{\gamma+1}{\gamma-1}\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{2\gamma}} - \frac{2}{\gamma-1} \qquad \text{(Wave Mach number)}$$

$$M = \frac{2}{\gamma-1}\left(1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{2\gamma}}\right)$$

(ii) Steep Flow

$$M_w = \left(\frac{r-1}{2r} + \frac{r+1}{2r} \times \frac{P_2}{P_1}\right)^{\frac{1}{2}} \qquad \text{(Wave mach number)}$$

$$M = \frac{{P_2}/{P_1} - 1}{\left(r\left(\frac{r-1}{2}\right)\frac{P_2}{P_1}\left(\frac{r+1}{r-1} + \frac{P_2}{P_1}\right)\right)^{\frac{1}{2}}}$$

AREA RATIO AS FUCTION OF MACH NUMBER

We know that

$$\textit{mass flow rate } m = \rho AC = \rho * A * C*$$

$$\frac{A}{A^*} = \frac{\rho *}{\rho} \times \frac{C *}{C}$$

$$M^{*2} = \frac{\left(\frac{\gamma + 1}{2}\right) M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M^2}$$

$$M^* = \left[\frac{\left(\frac{\gamma + 1}{2}\right) M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M^2}\right]^{\frac{1}{2}}$$

$$\frac{C}{C *} = \left[\frac{\left(\frac{\gamma + 1}{2}\right) M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M^2}\right]^{\frac{1}{2}} \qquad \left[M * = \frac{C}{a *}\right] \quad , [c * = a *]$$

$$\frac{C *}{C} = \frac{1}{\left[\frac{\left(\frac{\gamma + 1}{2}\right) M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M^2}\right]^{\frac{1}{2}}}$$

$$= \left[\frac{1 + \left(\frac{\gamma - 1}{2}\right) M^2}{\left(\frac{\gamma + 1}{2}\right) M^2}\right]^{\frac{1}{2}}$$

Multiply by 2 on RHS

$$\frac{C *}{C} = \left[\frac{2 + (\gamma - 1) M^2}{(\gamma + 1) M^2}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2}{(\gamma + 1) M^2} + \frac{(\gamma - 1)}{(\gamma + 1)}\right]^{\frac{1}{2}}$$

Taking $\frac{1}{M^2}$ *out*

$$\frac{C*}{C}=\frac{1}{M^2}\left[\frac{2}{(\gamma+1)}+\left(\frac{\gamma-1}{\gamma+1}\right)M^2\right]^{\frac{1}{2}}$$

Substituting

$$\frac{\rho*}{\rho} \text{ and } \frac{C*}{C}$$

$$\frac{A}{A^*}=\frac{\rho*}{\rho}\times\frac{C*}{C}$$

$$\frac{A}{A^*}=\frac{1}{M}\left[\frac{2}{(\gamma+1)}+\left(\frac{\gamma-1}{\gamma+1}\right)M^2\right]^{\frac{1}{\gamma-1}}\times\frac{1}{M}\left[\frac{2}{(\gamma+1)}+\left(\frac{\gamma-1}{\gamma+1}\right)M^2\right]^{\frac{1}{2}}$$

$$\frac{A}{A^*}=\frac{1}{M}\left[\frac{2}{(\gamma+1)}+\left(\frac{\gamma-1}{\gamma+1}\right)M^2\right]^{\frac{1}{\gamma-1}+\frac{1}{2}}$$

$$\frac{A}{A^*}=\frac{1}{M}\left[\frac{2}{(\gamma+1)}+\left(\frac{\gamma-1}{\gamma+1}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

We know that

$$\frac{P_o}{P}=\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{A}{A^*}\times\frac{P}{P_0}=\frac{1}{M}\left[\frac{2}{(\gamma+1)}+\left(\frac{\gamma-1}{\gamma+1}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}\times\frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{A}{A^*}\times\frac{P}{P_0}=\frac{\frac{1}{M}\left[\frac{2}{(\gamma+1)}+\left(\frac{\gamma-1}{\gamma+1}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$=\frac{\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left[\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}\right]}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$=\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\times\frac{\left[\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}\right]}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$=\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\times\left[\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}\right]\times\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{-\gamma}{\gamma-1}}$$

$$=\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\times\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}-\frac{-\gamma}{\gamma-1}}$$

$$=\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\times\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{-1}{2}}$$

$$\frac{A}{A^*}\times\frac{P}{P_0}=\frac{\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{1}{2}}}$$

IMPULSE FUCTION OR WALL FORCE FUCTION

The sum of pressure force (PA) and Impulse force (ρAC^2) gives

Impulse function (F)

Impulse function = PA+ ρAC^2

we know that for perfect gas

$PV = RT$

$$\frac{P}{\rho} = RT \qquad \left(\rho = \frac{1}{C}\right)$$

$$\rho = \frac{P}{RT}$$

$\times$ *by* C^2 *in the above equations*

$$\rho C^2 = \frac{PC^2}{RT}$$

$$\rho C^2 = \frac{\gamma PC^2}{\gamma RT} \qquad (a = \sqrt{\gamma RT})$$

$$= \frac{\gamma PC^2}{a^2}$$

$$\rho C^2 = \gamma PM^2$$

sub ρC^2 In Impulse function = $PA + \rho AC^2$

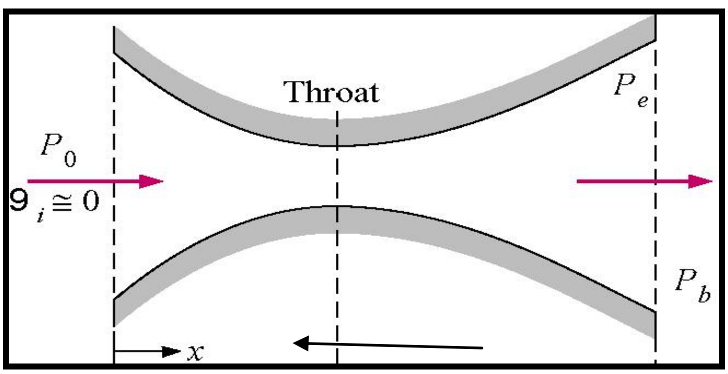
$F = PA + A \times \gamma PM^2$

$F = PA + \left[1 + \gamma M^2\right]$

one dimensional flow through a symmetrical stight duct is shown in fig below .The thrust or wall force

At sonic conditions M =1

At M=1 F=F*



IMPULSE FUCTION

$$\frac{\mathrm{F}}{\mathrm{F}*}=\frac{\mathrm{PA}+\left[1+\gamma \mathrm{M}^2\right]}{\mathrm{P}*\mathrm{A}*+\left[1+\gamma\right]}$$

$$\frac{\mathrm{F}}{\mathrm{F}*}=\frac{P}{P*}\times\frac{A}{A*}\times\frac{\left[1+\gamma \mathrm{M}^2\right]}{\left[1+\gamma\right]}$$

we know that

$$\frac{P}{P*}=\frac{1}{M}\frac{1}{\left[\frac{2}{\gamma-1}+\frac{\gamma-1}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{A}{A*}=\frac{1}{M}\left[\frac{2}{\gamma-1}+\frac{-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Sub $\frac{P}{P*}$ *and* $\frac{A}{A*}$

$$\frac{\mathrm{F}}{\mathrm{F}*}=\frac{1}{\left[\frac{2}{\gamma-1}+\frac{\gamma-1}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}}\times\frac{1}{M}\left[\frac{2}{\gamma-1}+\frac{-1}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\times\frac{\left[1+\gamma \mathrm{M}^2\right]}{\left[1+\gamma\right]}$$

$$=\frac{1}{M}\left[\frac{2}{\gamma-1}+\frac{\gamma-1}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}-\frac{\gamma}{\gamma-1}}\times\frac{\left[1+\gamma \mathrm{M}^2\right]}{\left[1+\gamma\right]}$$

$$=\frac{1}{M}\left[\frac{2}{\gamma-1}+\frac{\gamma-1}{\gamma+1}\right]^{-\frac{1}{2}}\times\frac{\left[1+\gamma \mathrm{M}^2\right]}{\left[1+\gamma\right]}$$

$$=\frac{1+\gamma \mathrm{M}^2}{M\left(1+\gamma\right)\left[\frac{2}{\gamma-1}+\frac{\gamma-1}{\gamma+1}\right]^{\frac{1}{2}}}$$

$$=\frac{1+\gamma \mathrm{M}^2}{M\left[\frac{2\left(1+\gamma\right)^2}{\gamma+1}\left[1+\frac{2}{\gamma-1}M^2\right]\right]^{\frac{1}{2}}}$$

$$= \frac{1+\gamma M^2}{M\left[2(\gamma+1)\left[1+\frac{2}{\gamma-1}M^2\right]\right]^{\frac{1}{2}}}$$

$$\frac{F}{F*} = \frac{1+\gamma M^2}{M\sqrt{2(\gamma+1)\left[1+\frac{2}{\gamma-1}M^2\right]}}$$

Another nondimensional expression for impulse fuction

$$\frac{F}{P_0 A*} = \frac{PA+\left[1+\gamma M^2\right]}{P_0 A*} \qquad \left[F = PA+\left[1+\gamma M^2\right]\right]$$

$$\frac{F}{P_0 A*} = \frac{P}{P_0} \times \frac{A}{A*}\left[1+\gamma M^2\right]$$

we know that

$$\frac{P}{P_0} = \left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{A}{A*} = \frac{1}{M}\left[\frac{2}{\gamma-1}+\frac{-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{P_0}{P*} \quad and \quad \frac{A}{A*}$$

$$\frac{F}{P_0 A*} = \frac{1}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}} \times \frac{1}{M}\left[\frac{2}{\gamma-1}+\frac{-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\gamma M^2\right]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma-1}+\frac{-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{-\gamma}{\gamma-1}} \times \left[1+\gamma M^2\right]$$

After simplying we get

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}-\frac{\gamma}{\gamma-1}} \times \left[1+\gamma M^2\right]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{-1}{2}} \times \left[1+\gamma \mathrm{M}^2\right]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{\left[1+\gamma \mathrm{M}^2\right]}{\sqrt{1+\frac{\gamma-1}{2}M^2}}$$

$$\frac{F}{P_0 A*} = \left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{\left[1+\gamma \mathrm{M}^2\right]}{M\sqrt{1+\frac{\gamma-1}{2}M^2}}$$

MASS FLOW RATE IN TERMS OF PRESSURE RATIO

from continuity equation the mass flow rate is

$$\dot{m} = \rho A C$$

For Stagnation pressure and stagnation density releation

$$\frac{\rho_0}{\rho} = \left[\frac{p_0}{P}\right]^{\frac{1}{\gamma}}$$

$$\rho = \frac{\rho_0}{\left[\frac{p_0}{P}\right]^{\frac{1}{\gamma}}}$$

$$\rho = \rho_0\left[\frac{p_0}{P}\right]^{-\frac{1}{\gamma}} = \rho_0\left[\frac{p}{P_0}\right]^{\frac{1}{\gamma}}$$

Stagnation temperature

$$T_0 = T + \frac{C^2}{2C_P} \qquad \left(C_P = \frac{\gamma R}{\gamma-1}\right)$$

$$C^2 = 2\frac{\gamma R}{\gamma-1}\left(T_0 - T\right)$$

$$C^2 = 2\frac{\gamma R}{\gamma - 1} T_0 \left(1 - \frac{T}{T_0}\right)$$

$$C^2 = 2\frac{\gamma R}{\gamma - 1} T_0 \left(1 - \left[\frac{p}{P_0}\right]^{\frac{\gamma-1}{\gamma}}\right) \qquad \left[\frac{T}{T_0} = \left[\frac{p}{P_0}\right]^{\frac{\gamma-1}{\gamma}}\right]$$

$$C = \sqrt{2\frac{\gamma}{\gamma - 1} RT_0 \left(1 - \left[\frac{p}{P_0}\right]^{\frac{\gamma-1}{\gamma}}\right)}$$

rom continuity equation the mass flow rate is
sub ρ *and* C

$$\dot{m} = \rho A C$$

$$\dot{m} = \rho_0 \left[\frac{p}{P_0}\right]^{\frac{1}{\gamma}} \times A \times \sqrt{2\frac{\gamma}{\gamma - 1} RT_0 \left(1 - \left[\frac{p}{P_0}\right]^{\frac{\gamma-1}{\gamma}}\right)}$$

We know that

$$\rho_0 = \frac{P_0}{RT_0}$$

$$\dot{m} = \frac{P_0}{RT_0} \times \left[\frac{p}{P_0}\right]^{\frac{1}{\gamma}} \times A \times \sqrt{2\frac{\gamma}{\gamma - 1} RT_0 \left(1 - \left[\frac{p}{P_0}\right]^{\frac{\gamma-1}{\gamma}}\right)}$$

$$= \frac{AP_0}{\sqrt{RT_0 \times \sqrt{RT_0}}} \times \left[\frac{p}{P_0}\right]^{\frac{1}{\gamma}} \times \sqrt{RT_0} \times \sqrt{2\frac{\gamma}{\gamma - 1} RT_0 \left(1 - \left[\frac{p}{P_0}\right]^{\frac{\gamma-1}{\gamma}}\right)}$$

$$= \frac{AP_0}{\sqrt{RT_0}} \times \sqrt{2\frac{\gamma}{\gamma - 1} \left(\left[\frac{p}{P_0}\right]^{\frac{2}{\gamma}} - \left[\frac{p}{P_0}\right]^{\frac{2}{\gamma} + \frac{\gamma+1}{\gamma}}\right)}$$

$$\dot{m} = \frac{AP_0}{\sqrt{RT_0}} \times \sqrt{2\frac{\gamma}{\gamma - 1} \left(\left[\frac{p}{P_0}\right]^{\frac{2}{\gamma}} - \left[\frac{p}{P_0}\right]^{\frac{2}{\gamma} + \frac{\gamma-1}{\gamma}}\right)}$$

$$\frac{\dot{m}}{A}=\frac{\mathrm{AP}_0}{\sqrt{RT_0}}\times\sqrt{2\frac{\gamma}{\gamma-1}\left(\left[\frac{\mathrm{p}}{\mathrm{P}_0}\right]^{\frac{2}{\gamma}}-\left[\frac{\mathrm{p}}{\mathrm{P}_0}\right]^{\frac{\gamma+1}{\gamma}}\right)}$$

$$\frac{\dot{m}}{A}=\frac{\mathrm{AP}_0}{\sqrt{RT_0}}\times\sqrt{\gamma}\times\sqrt{\frac{2}{\gamma-1}\left(\left[\frac{\mathrm{p}}{\mathrm{P}_0}\right]^{\frac{2}{\gamma}}-\left[\frac{\mathrm{p}}{\mathrm{P}_0}\right]^{\frac{\gamma+1}{\gamma}}\right)}$$

$$\frac{\dot{m}\sqrt{RT_0}}{\mathrm{AP}_0\sqrt{\gamma}}=\sqrt{\frac{2}{\gamma-1}\left(\left[\frac{\mathrm{p}}{\mathrm{P}_0}\right]^{\frac{2}{\gamma}}-\left[\frac{\mathrm{p}}{\mathrm{P}_0}\right]^{\frac{\gamma+1}{\gamma}}\right)}$$

$$\frac{\dot{m}\times\sqrt{T_0}}{\mathrm{AP}_0}\times\sqrt{\frac{R}{\gamma}}=\sqrt{\frac{2}{\gamma-1}\times\left(\left[\frac{\mathrm{p}}{\mathrm{P}_0}\right]^{\frac{2}{\gamma}}-\left[\frac{\mathrm{p}}{\mathrm{P}_0}\right]^{\frac{\gamma+1}{\gamma}}\right)}$$

This equation gives mass flow rate interms of pressure ratio

For maximum flow rate condition $m = m_{max}$ and $A = A^*$

We know that

$$\left(\frac{p}{p_0}\right)_{\max}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma+1}}$$

$$sub\ \left(\frac{p}{p_0}\right)$$

$$\frac{\dot{m}_{\max}\times\sqrt{T_0}}{\mathrm{A^*P}_0}\times\sqrt{\frac{R}{\gamma}}=\sqrt{\frac{2}{\gamma-1}\times\left(\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma+1}}-\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}\times\frac{\gamma+1}{\gamma}}\right)}$$

$$=\sqrt{\frac{2}{\gamma-1}\times\left(\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma+1}}-\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma 11}}\right)}$$

$$mulplying\quad and\ \ divide\ by\ \ \frac{2}{\gamma+1}$$

$$= \sqrt{\frac{2}{\gamma - 1} \times \left(\left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{2}{\gamma + 1} \right) \times \left(\frac{\gamma + 1}{2} \right) - \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \right)}$$

$$= \sqrt{\frac{2}{\gamma - 1} \times \left(\left(\frac{2}{\gamma + 1} \right)^{\frac{2+\gamma-1}{\gamma-1}} \times \left(\frac{\gamma + 1}{2} \right) - \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \right)}$$

$$= \sqrt{\frac{2}{\gamma - 1} \times \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \times \left[\frac{\gamma + 1}{2} - 1 \right]}$$

$$= \sqrt{\frac{2}{\gamma - 1} \times \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \times \left[\frac{\gamma + 1 - 2}{2} \right]}$$

$$= \sqrt{\frac{2}{\gamma - 1} \times \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \times \left[\frac{\gamma - 1}{2} \right]}$$

$$= \sqrt{\left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$= \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

$$\frac{\dot{m}_{\max} \times \sqrt{T_0}}{\mathrm{A^*P_0}} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$\gamma = 1.4$

R=287 J / KgK

$$\frac{\dot{m}_{\max} \times \sqrt{T_0}}{\mathrm{A^*P_0}} \times \sqrt{\frac{287}{1.4}} = \left(\frac{2}{1.4 + 1} \right)^{\frac{1.4+1}{2(1.4-1)}}$$

$$\frac{\dot{m}_{\max} \times \sqrt{T_0}}{\mathrm{A^*P_0}} = 0.0404$$

MASS FLOW RATE IN TERMS OF AREA RATIO

$$\textit{mass flow rate } m = \rho AC = \rho * A * C*$$

$$\div \ \ by \ A$$

$$\frac{m}{A} = \rho C = \rho * \frac{A*}{A} C*$$

We know that

$$\rho* = \frac{P*}{R*T*}$$

$$\frac{m}{A} = \frac{P*}{R*T*} \times \frac{A*}{A} \times C* \left(C* = a* = \sqrt{\gamma RT*} \right)$$

$$\frac{m}{A} = \frac{P*}{R*T*} \times \frac{A*}{A} \times \sqrt{\gamma RT*}$$

$$\frac{m}{A} = \frac{P*}{\sqrt{T*}} \times \frac{A*}{A} \times \sqrt{\frac{\gamma}{R}}$$

$$\frac{T_0}{T*} = \frac{\gamma + 1}{2}$$

$$T* = \left(\frac{2}{\gamma + 1} \right) T_0$$

$$\frac{P_0}{P*} = \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma}{\gamma - 1}}$$

$$P* = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} P_0$$

$$\frac{m}{A} = \frac{A*}{A} \times \frac{\left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} P_0}{\sqrt{\left(\frac{2}{\gamma + 1} \right) T_0}} \times \sqrt{\frac{\gamma}{R}}$$

$$\frac{m}{A} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \sqrt{\left(\frac{\gamma + 1}{2} \right)} \times \frac{P_0}{\sqrt{T_0}} \times \sqrt{\frac{\gamma}{R}} \times \frac{A*}{A}$$

$$\frac{m\sqrt{T_0}}{AP_0}\sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{2}{\gamma+1}\right)^{-\frac{1}{2}} \times \frac{A*}{A}$$

$$\boxed{\frac{m\sqrt{T_0}}{AP_0}\sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{A*}{A}}$$

$$for\, max\ mass\ \ flow\ rate\ \ condition,\ m = m_{\max\ and\ \ A=A*}$$

$$\frac{m_{\max}\sqrt{T_0}}{A*P_0}\sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

MASS FLOW RATE IN TERMS OF MACH NUMBER

From Equation , we know that

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left[1+\frac{\gamma-1}{2}M^2\right]\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Substituting $\frac{A}{A^*}$

$$\frac{m\sqrt{T_0}}{A\,p_0} \times \sqrt{\frac{R}{\gamma}} = \frac{\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}{\frac{1}{M}\left[\frac{2}{\gamma+1}\left[1+\frac{\gamma-1}{2}M^2\right]\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times M}{\left[\left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left[1+\frac{\gamma-1}{2}M^2\right]\right]^{\frac{\gamma+1}{2(\gamma-1)}}\right]}$$

$$= \frac{M}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\boxed{\frac{m\sqrt{T_0}}{A\,p_0} \times \sqrt{\frac{R}{\gamma}} = \frac{M}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}}$$

This equation gives the mass flow rate in terms of Mach number for $m$ax *mass flow rate condition,* $m = m_{\max\ and\ A=A^*}$

$M = 1$

$$\frac{m\sqrt{T_0}}{A^*\,p_0} \times \sqrt{\frac{R}{\gamma}} = \frac{1}{\left[1+\frac{\gamma-1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{1}{\left[\frac{2+\gamma-1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{1}{\left[\frac{\gamma+1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\frac{m\sqrt{T_0}}{A^*\, p_0} \times \sqrt{\frac{R}{\gamma}} = \frac{1}{\left[\frac{\gamma+1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Q 8) A conical diffuser has entry diameter 20 cm, the match number temp and pressure are 0.6, 120 KN/m^2 and 340 K. The mach number at exist is 0.2. For 1 - D isentropic flow, calculate the following.

i) Pressure, temp and velocity at exist

ii) Mass flow rate, and exit diameter

iii) Change in impulse function

From the gas tables,

$$\left.\frac{T_1}{T_o}\right]_{M=0.6} = 0.933$$

$$T_o = \frac{340}{0.933} = 364.4 \text{ K}$$

Stagnation temperature T_o = 364.4 K

$$\left.\frac{P_1}{P_o}\right]_{M=0.6} = 0.784$$

$$P_o = \frac{1.2 \times 10^5}{0.784}$$

$P_o = 1.53 \times 10^5$ N/m^2

To find temperature at the exit

We have,

$$\left.\frac{T_2}{T_o}\right]_{M=0.2} = 0.992$$

$T_2 = 364.4 \times 0.992$

$T_2 = 361.5$ K

To find pressure at the exit

We have,

$$\left.\frac{P_2}{P_o}\right]_{M=0.2} = 0.973$$

$P_2 = 1.53 \times 10^5 \times 0.973$

$= 1.48 \times 10^5$ N/m^2

To find velocity at the exit

Sonic velocity at (1)

$C_2 = \sqrt{KRT_2} = 20.05\ \sqrt{361.5}$

$C_2 = 381.2$ m/s

Velocity $V_2 = M_2 \times C_2$

$= 0.2 \times 381.2$

$V_2 = 76.2$ ms^{-1}

To find mass flow rate

Velocity at inlet

$V_1 = C_1 \times M_1 = .\ \sqrt{KRT_1}\ \times M_1$

$= 20.05\ \sqrt{340} \times 0.6$

$V_1 = 221.82$ ms^{-1}

$r_1 = \dfrac{P_1}{RT_1} = \dfrac{1.2 \times 10^5}{287 \times 340} = 1.23$ Kg/m^3

Mass flow rate = r_1 A V_1

$\dot{m} = 1.23 \times \dfrac{\pi}{4} (0.2)^2 \times 221.82 = 8.57$ Kg/s

To find the diameter (d_2)

We have $\left.\frac{A_2}{A^*}\right]_{0.2}$ = 2.914

$\left.\frac{A_1}{A^*}\right]_{0.6}$ = 1.188

$$\frac{A_2}{A_1} = \frac{2.964}{1.188} = \frac{(d_2)^2}{(d_1)^2}$$

$$d_2 = \left[\frac{2.964 \times 20^2}{1.188}\right]^{1/2} = 31.5 \text{ cm}$$

$\therefore$ Exit diameter, d_2 = 31.5 cm

Impulse function

$$F = PA + rAV^2 = PA\left[1+\frac{\rho}{P}V^2\right]$$

$$= PA\left[1+\frac{1}{RTP}V^2\right] = PA\left[1+\frac{KV^2}{KRT}\right]$$

i.e., $F = PA\left[1+KM^2\right]$

$DF = F_2 - F_1$

$DF = P_1A_1\left[1+KM_1^2\right] - P_2A_2\left[1+KM_2{}^2\right]$

$F1 = P_1A_1\left[1+KM_1^2\right]$

$$= 1.2 \times 10^5 \times \frac{\pi}{4}(0.2)^2\left[1+1.4\times(0.6)^2\right] = 5.667 \text{ KN}$$

$= P_2A_2\left[1+KM_2^2\right]$

$$= 1.48 \times 10^5 \times \frac{\pi}{4}(0.315)^2\left[1+1.4\times(0.2)^2\right]$$

= 12.17 KN

$DF = F_2 - F_1$ = 12.17 - 5.667

DF = 6.503 KN

Q 9) Air is discharged from a reservoir at Po = 6.91 Kgf/cm^2 and to = 325°C through a nozzle to an exit pressure of 0.98 Kgf/cm^2 . If the flow rate is 1 kg/s, find the throat area, pressure and velocity. Also find the exit area, exit temperature and exit velocity.

Ans

Exit Mach number M_e] = 1.93

$$\frac{P_e}{P_o} = \frac{0.98}{6.91} = 0.142$$

To find exit temperature

$$\left.\frac{T_e^*}{T_o}\right]_{M=1.93} = 0.573$$

$T_e = 598 \times 0.573$

= 342.65K

$$r_e = \frac{P_e}{R\,T_e} = \frac{0.98 \times 9.81 \times 10^4}{287 \times 342.65}$$

$r_e = 0.975\ Kg/m^3$

To find velocity at exit (V_e)

Sonic Velocity

$$C_e = \sqrt{KRT_e} = 20.05\sqrt{342.65}$$

$C_e = 371.14\ ms^{-1}$

$V_e = M_c\, C_e = 1.93 \times 371.14 = 716.3\ ms^{-1}$

To find exit area

We have

$\dot{m} = r_e\, A_e\, V_e$

$$A_e = \frac{\dot{m}}{\rho_e V_e} = \frac{1}{0.978 \times 716.3} = \text{......}\ 0.0014\ m^2$$

To find throat pressure (P*)

$$\left.\frac{P^*}{P_o}\right]_{M=1} = 0.528$$

$P^* = 0.528 \times 6.91 = 3.65$ K gf/cm^2

To find throat tempeature (T*)

$$\left.\frac{T^*}{T_o}\right]_{M=1} = 0.834$$

$T^* = 0.834 \times 598 = 498.732$ K

$$\left.\frac{A}{A^*}\right]_{M=1} = 1.000$$

$$\left.\frac{A}{A^*}\right]_{M=1.93} = 1.593$$

$$A^* = \frac{A_e}{1.593} = \frac{0.0014}{1.593} = 0.00088\ m^2$$

$$V^* = \sqrt{KRT^*} = 20.05\sqrt{498.732} = 447.76\ ms^{-1}$$

Q 10) A supersonic wind tunnel is to be designed to give a mach number 2 (with air) T the test section, having an area of 0.1 m^2. The air pressure and temperature at inlet to the nozzle where the velocity is negligible are 5×10^5 N/m^2 and 150^{o}C. Find the nozzle throat area, pressure and temperature at test section and mass flow rate.

Ans

To find throat area

$$\left.\frac{A}{A^*}\right]_{M=2} = 1.687$$

$$A^* = \frac{0.1}{1.687} M^2 = 0.06\ M^2$$

$$\left.\frac{T_e}{T_o}\right]_{M=2} = 0.555$$

Exit temperature,

$T_e = T_o \times 0.555 = 423 \times 0.555$

$T_e = 234.77\ K$

Exit pressure,

$$\left.\frac{P_e}{P_o}\right]_{M=2} = 0.128$$

$P_e = P_o \times 0.128 = 5 \times 10^5 \times 0.128$

$P_e = 0.64 \times 10^5\ N/m^2$

To find mass flow rate

Mass flow rate,

$\dot{m} = r_e\ A_e\ V_e$

$$r_e = \frac{P_e}{RT_e} = \frac{0.64 \times 10^5}{287 \times 234.77}$$

$= 0.9498 Kg/\ m^3$

$V_e = M_e \times C_e = 2\sqrt{KRT_e} = ..2 \times 20.05\ \sqrt{234.77}$

$V_e = 614.42\ ms^{-1}$

Mass flow rate

$\dot{m} = r_e\ A_e\ V_e$

$= 0.9498 \times 0.1 \times 614.42$

$\dot{m} = 58.36\ kg/s$

Q 11) Air flows isentropically through a convergent-divergent nozzle. The inlet stagnation conditons are 700 KN/m^2 and 320^oC. The exist pressure is 10^5 KN/m^2 and exit area is 6.25 cm^2. Find

i) Mach number, temp and velocity at exit

ii) Pressure, velocity and temperature at the throat

iii) Mass flow rate

iv) Throat area

Ans.

To find properties at exit

Mach number (M_e)

$$M_e\Big]_{\frac{P_e}{P_o}=\frac{1.05}{7}=0.15} = 1.90$$

Temperature

$$\frac{T_e}{T_o}\Bigg]_{M=1.90} = 0.581$$

$T_c = 593 \times 0.581 = 344.5$ K

Sonic velocity

$C_e = \sqrt{KRT_e} = 20.05 \sqrt{344.5}$

$C_e = 372.16$ ms^{-1}

Velocity,

$V_e = M_e \times C_e = 1.9 \times 372.16 = 707.1$ ms^{-1}

Throat properties

Pressure

$$\frac{P*}{P_o}\Bigg]_{M=1} = 0.528$$

$P* = 7 \times 10^5 \times 0.528 = 3.696 \times 10^5$ N/m^2

Temperature

$$\left.\frac{T^*}{T_o}\right]_{M=1} = 0.834$$

T* = 593 × 0.834

= 494.6 K

Velocity (V*)

V* = C* = $\sqrt{KRT^*}$

= 20.05 $\sqrt{494.6}$

V* = 445.89 ms^{-1}

Throat area

$$\left.\frac{A_c}{A^*}\right]_{M=1.9} = 1.555$$

$$A^* = \frac{6.25 \times 10^{-4}}{1.555}$$

A* = 4.019 × 10^{-4} m^2

Mass flow rate

$$r_e = \frac{P_e}{RT_e} = \frac{1.05 \times 10^5}{287 \times 344.5}$$

= 1.06 Kg/m^3

Mass flow rate $(\dot{m})_2 = r_e A_e V_e$

$= 1.06 \times 6.25 \times 10^{-4} \times 707.1$

$\dot{m} = 0.4684$ Kg/s

Q 12) Air at 60^{o}C at 1.1 bar enters a insulated passage with M = 0.4. Air is expanded isentropically until its pressure is 1.05 bar. Find

i) Final mach number.

ii) Final temperature, density and velocity

iii) Area ratio of the passge

To find final temperature (T_2)

$$\left.\frac{T_1}{T_o}\right]_{M=0.4} = 0.969$$

$$T_o = \frac{333}{0.969}$$

= 343.65 K

$$\left.\frac{P_1}{P_o}\right]_{M=0.4} = 0.895$$

$$P_o = \frac{1.1}{0.895} = 1.229\,\text{bar}$$

$$M_2\Big]_{\frac{P_2}{P_o}=\frac{1.05}{1.229}=0.854} = 0.48$$

Mach number at exit (M_2) = 0.48

$$\left.\frac{T_2}{T_o}\right]_{M_2=0.48} = 0.956$$

$T_2 = T_o \times 0.956$

= 343.65 × 0.956 = 328.53 K

$\therefore$ Final temperature T_2 = 328.53 K

Final density $r_2 = \dfrac{P_2}{RT_2}$

$$= \frac{1.05 \times 10^5}{287 \times 328.53} = 1.11$$

Final density r_2 = 1.11 Kg/m^3

Sonic velocity

$C^2 = \sqrt{KRT_2}$

$= 20.05\ \sqrt{328.53}$

$= 363.41\ ms^{-1}$

Final velocity

$V_2 = C_2 \times M_2$

$= 363.41 \times 0.48$

$= 174.44\ ms^{-1}$

To find the area ratio of the passage

$$\left.\frac{A_1}{A*}\right]_{M_1=0.4} = 1.59$$

$$\left.\frac{A_2}{A*}\right]_{M_2=0.48} = 1.38$$

Area ratio = $\frac{A_1}{A_2} = \frac{1.59}{1.38} = 1.152$

Q 13) Air flows isentropically through a C.D. The inlet conditions are pressure 700 KN/m^2, temperature 320^{o}C, velocity 50 m/s. The exit pressure is 105 KN/m^2 and the exist area is 6.25 cm^2. Calculate

i) Mach number, temperature and velocity at exit

ii) Pressure, temperature and velocity at throat

iii) Mass flow rate

iv) Throat area

Ans: To find the exit mach number (M_2)

Sonic velocity at inlet

$C_1 = \sqrt{KRT_1}$

$= 20.05\ \sqrt{593}$

$= 488.25 \text{ ms}^{-1}$

Inlet mach number

$M_1 = \frac{V_1}{C_1}$

$= \frac{50}{488.25} = 0.1$

We have

$\left.\frac{P_1}{P_o}\right]_{M=0.1} = 0.993$

$P_o = \frac{700}{3.993}$

$= 7.04 \times 10^5 \text{ N/m}^2$

Exit mach number

$M_2\Big]_{\frac{P_2}{P_o}=\frac{105}{704}=0.149} = 1.9$

We have

$\left.\frac{T_1}{T_o}\right]_{M=0.1} = 0.998$

$To = \frac{593_1}{0.998}$

$= 594.2 \text{ K}$

We have

$\left.\frac{T_2}{T_o}\right]_{M_2=1.9} = 0.581$

$T_2 = 594.2 \times 0.581$

Exit temperature $T_2 = 345.22$ K

Sonic velocity

$C_2 = \sqrt{KRT_2}$

$= 20.05\ \sqrt{345.22}$

$= 372.5\ ms^{-1}$

Final velocity

$V_2 = M_2 \times C_2$

$= 1.9 \times 372.5$

$= 707.8\ ms^{-1}$

To find throat parameters

$$\left.\frac{P*}{P_o}\right]_{M=1} = 0.528$$

Throat pressure

$P* = 7.04 \times 10^5 \times 0.528$

$P* = 371.7\ KN/m^2$

$$\left.\frac{T*}{T_o}\right]_{M=1} = 0.834$$

Throat temperature

$T* = 594.2 \times 0.834$

$T* = 495.5\ K$

Velocity

$V* = C*$

$= \sqrt{KRT*}$

$= 20.05\ \sqrt{495.5}$

Throat velocity $V* = 446.3\ ms^{-1}$

$$\left.\frac{A_2}{A*}\right]_{M=1.9} = 1.555$$

$A^* = \frac{6.25 \times 10^{-4}}{1.555}$

Throat area

$A^* = 4.019 \times 10^{-4}$ m^2

Q 14) Air is discharged from a C - D nozzle. Pilot-tube readings at inlet and exist of the nozzle ae 6.95 × 105 N/m^2 and 5.82 × 10^5 N/m^2 respectively. The inlet stagnation temp 250^{o}C and exit static pressure, 1.5 × 10^5 N/m^2. Find the inlet and exit stagnation pressure, exit Mach numbers, exit flow velocity and nozzle efficiency.

Note

i) (Pitot - tube reading gives stagnation pressure)

ii) (Since stagnation pr. vlaues are different for inlet and exit the flow is no longer isentropic)

Mach number at exist (M_a)

Consider the isentropic deceleration process shown (a - oa)

$$M_a\Big]_{\frac{P_a}{P_{oa}} = \frac{1.5\times10^5}{5.82\times10^5} = 0.257} = 1.54$$

Exit flow velocity (V_a)

$$Ma = \frac{V_a}{C_a}$$

$$Ca = \sqrt{KRT_a}$$

$$\frac{T_a}{T_{oa}}\Big]_{M=1.54} = 0.678$$

$T_a = 6.23 \times 0.678$

$= 422.4$ K

$C_a = 20.05\ \sqrt{422.4}$

$= 412.1$ ms^{-1}

$V_a = M_a \times C_a$

$= 412.1 \times 1.54$

$= 634.6\ ms^{-1}$

Nozzle efficiency (h_N)

$$h_N = \frac{T_o - T_a}{T_o - T_i}$$

To get T_i

For the isentropic i - oi

$$M_i \Big]_{\frac{P_i}{P_{oi}} = \frac{1.5\times10^5}{6.75\times10^5} = 0.222}$$

$$\frac{T_i}{T_{oi}}\Bigg]_{M=1.64} = 0.650$$

$T_i = 623 \times 0.65$

$= 404.95\ K$

Nozzle efficiency

$$(h_N) = \frac{T_o - T_a}{T_o - T_i} \times 100$$

$$= \left(\frac{623 - 422.4}{623 - 404.95}\right) \times 100$$

$h_N = 91.99\ \%$

Q 15) A nozzle has an exit to throat area ration of 2.5. The reservoir conditions ae 7 Kg/cm^2 and 60^oC. The nozzle efficiency is 90%. The flow may be assumed to be isentropic up to the throat. Calculate the pressure velocity and mach number at the exit and compare them with the corresponding values for isentropic flow.

For isentropic floe (o – i)

$$M_i \Big]_{\frac{A_i}{A^*} = 2.5 = 2.44}$$

$$\left.\frac{T_i}{T_o}\right]_{M=2.44} = 0.456$$

$T_i = 333 \times 0.456 = 151.84$ K

$$\left.\frac{P_i}{P_o}\right]_{M=2.44} = 0.0643$$

$P_i = 0.0643 \times 7 = 0.45$ Kg/cm2

$C_i = \sqrt{KRT_i}$

$= 20.05\sqrt{151.84}$

$C_i = 247.1\ ms^{-1}$

$V_i = Mi \times Ci$

$= 2.44 \times 247.1$

$C_i = 602.8\ ms^{-1}$

For the actual flow (o – a)

Nozzle efficiency,

$$h_N = \frac{T_0 - T_a}{T_0 - T_i}$$

$$0.9 = \frac{333 - T_a}{333 - 151.84}$$

$T_a = 169.9$ K

$C_a = 20.05\sqrt{T_a} = 20.05\sqrt{169.9}$

$C_a = 261.2\ ms^{-1}$

Considering the isentropic flow (oa – a)

$$M_a\Big]_{\frac{T_a}{T_{oa}} = \frac{169.9}{333} = 0.510} = 2.19$$

$V_a = M_a \times C_a = 2.19 \times 261.4$

$V_a = 572.5 \text{ ms}^{-1}$

$P_a = P_i$ = 0.45 Kgf/cm^2

Property	Actual flow	Ideal Flow	% deviation
Mach number	2.19	2.44	10.2%
Pressure	0.45	0.45	0%
Velocity	572.5	602.8	5.02%
Temperature	169.9	151.84	10.6%

Q16) A converging nozzle operating with air at inlet conditions P_0 = 4 Kgf/cm^2. T_0 = 450^{o}C and T = 400^{o}C is expected to have an exit static pressure of 2.5 Kgf/cm^2. Under ideal conditions. Estimate the exit temperature and mach number, assuming a nozzle efficiency = 0.92 when the expansion takes places to the same back pressure.

Ans:

$$M_{2i}\Big]_{\frac{P_{2i}}{P_0}=\frac{2.5}{4}=0.625} = 0.85$$

$$\frac{T_{2i}}{T_o}\Bigg]_{M=0.85} = 0.874$$

T_{2i} = 723 × 0.874 = 631.9 K

Nozzle efficiency

$$h_N = \frac{T_1 - T_a}{T_1 - T_{2i}}$$

$$0.92 = \frac{673 - T_{2a}}{673 - 631.9}$$

$$T_{2a} = 635.2 \text{ K}$$

$$M_{2a}\Big]_{\frac{T_{2a}}{T_{02a}}=\frac{635.2}{723}} = 0.879$$

Q.17) A converging, nozzle operating with air and inlet conditions of P = 4 Kg/cm^2, T_0 = 450^{o}C and T = 400^{o}C is expected to have an exit static pressure of 2.5 Kgf/cm$_2$ under ideal conditions. Estimate the exit temperature and mach number, assuming a nozzle efficiency = 0.92 when the expansion takes place to the same back pressure.

$$M\Big]_{\frac{T}{T_0}=\frac{673}{723}=0.93} = 0.61$$

$$\frac{P}{P_0}\Big]_{M=0.61} = 0.778$$

$$P_0 = \frac{4}{0.778} = 5.14 \text{ Kgf} / \text{cm}^2$$

$$M_{2i}\Big]_{\frac{P_{2i}}{P_0}=\frac{2.5}{5.14}=0.486} = 1.07$$

$$\frac{T_{2i}}{T_0}\Big]_{M=1.07} = 0.814$$

$$T_{2i} = 723 \times 0.814 = 588.5 \text{ K}$$

Nozzle efficiency

$$h_N = \frac{T_1 - T_{2a}}{T_1 - T_{2i}}$$

$$0.92 = \frac{673 - T_{2a}}{673 - 588.5}$$

$$T_{2a} = 595.3 \text{ K}$$

$$M_{2a}\Big]_{\frac{p_{2a}}{p_{20}}=\frac{595.3}{7.23}=0.823} = 1.04$$

Q18) Air is supplied through a nozzle with an exit area 6 cm^2. A tank supplies air at 5 kg/cm^2 and 150^{o}C. The discharge pressure is 3 Kg/cm^2. Assume the co-efficient of velocity is 0.9, find the discharge velocity, mach number, mass flow rate and entropy increase.

$$M_i\Big]_{\frac{P_e}{P_0}=\frac{3}{5}=0.6} = 0.89$$

$$\left.\frac{T_{1i}}{T_0}\right]_{M=0.89} = 0.863$$

$$T_{1_i} = 423 \times 0.863 = 365.1 \text{ K}$$

$$C_i = \sqrt{KRT_i} = 20.05\sqrt{341.36} = 383.1 \text{ ms}^{-1}$$

$$V_i = M_i \times C_i = 1.26 \times 370.4 = 482.7 \text{ ms}^{-1}$$

Mach Number (M_a)

$$M_a = \frac{V_a}{C_a}$$

$$\frac{T_0 - T_a}{T_0 - T_1} = 0.9^2$$

$$\frac{423 - T_a}{423 - 365.1} = 0.9^2$$

$$T_a = 376.1 \text{ K}$$

$$M_a = \frac{390.97}{20.05\sqrt{376.1}}$$

$$M_a = 1.005$$

$$DS_{o-a} = C_p \ln\frac{T_a}{T_0} - R \ln\frac{P_a}{P_o}$$

or

$$DS_{o-a} = DS_{i-a} = C_p \ln\frac{T_a}{T_i} - R \ln\frac{P_a}{P_i} = C_p \ln\frac{T_a}{T_i}$$

$$= 1005 \ln\frac{376.1}{365.1} = 29.83 \text{ J/Kg K}$$

Module-3

GINO GIROLAMO FANNO

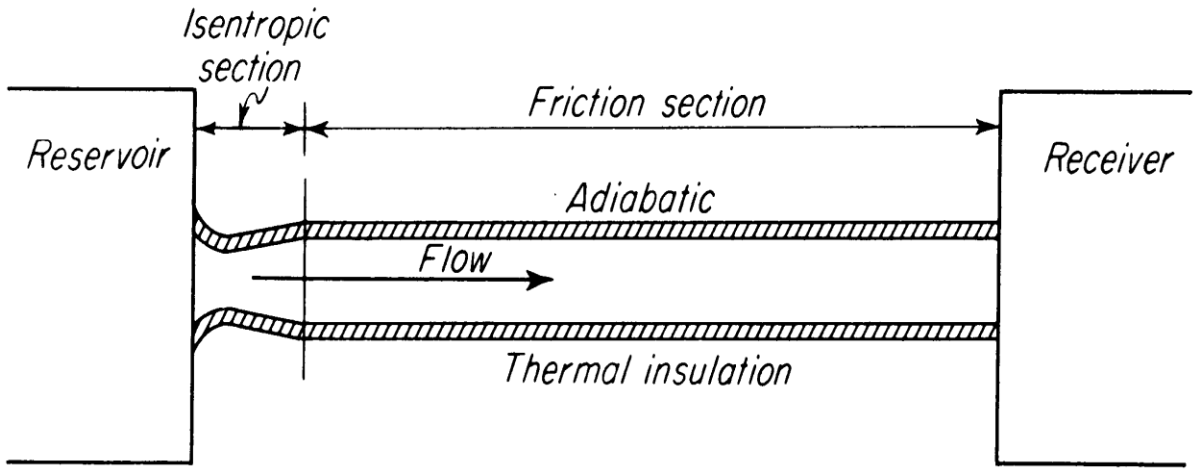
Fanno a Jewish Engineer was born on November 18, 1888. He studied in a technical institute in Venice and graduated with very high grades as a mechanical engineer. Fanno was not as lucky as his brother, who was able to get into academia. Faced with anti–Semitism, Fanno left Italy for Zurich, Switzerland in 1900 to attend graduate school for his master's degree. In this new place he was able to pose as a Roman Catholic, even though for short time he went to live in a Jewish home, Isaak Baruch Weil's family. As were many Jews at that time, Fanno was fluent in several languages including Italian, English, German,and French. He likely had a good knowledge of Yiddish and possibly some Hebrew.Consequently, he did not have a problem studying in a different language. In July 1904 he received his diploma (master). When one of Professor Stodola's assistants attended military service this temporary position was offered to Fanno. "Why didn't a talented guy like Fanno keep or obtain a position in academia after he published his model?" The answer is tied to the fact that somehow rumors about his roots began to surface. Additionally, the fact that his model was not a "smashing success" did not help

Later Fanno had to go back to Italy to find a job in industry. Fanno turned out to be a good engineer and he later obtained a management position. He married, and like his brother, Marco, was childless. He obtained a Ph.D. from Regian Istituto Superiore d'Ingegneria di Genova. However, on February 1939 Fanno was degraded (denounced) and he lost his Ph.D. (is this the first case in history) because his of his Jewish nationality51. During the War (WWII), he had to be under house arrest to avoid being sent to the "vacation camps." To further camouflage him self,Fanno converted to Catholicism. Apparently, Fanno had a cache of old Italian currency
(which was apparently still highly acceptable) which helped him and his wife survive the war. After the war, Fanno was only able to work in agriculture and agricultural

engineering. Fanno passed way in 1960 without world recognition for his model.Fanno's older brother, mentioned earlier Marco Fanno is a famous economist who later developed fundamentals of the supply and demand theory.

An adiabatic flow with friction is named after Ginno Fanno a Jewish engineer. This model is the second pipe flow model described here. The main restriction for this model is that heat transfer is negligible and can be ignored. This model is applicable to flow processes which are very fast compared to heat transfer mechanisms with small Eckert number. This model explains many industrial flow processes which includes emptying of pressured container through a relatively short tube, exhaust system of an internal combustion engine, compressed air systems, etc. As this model raised from need to explain the steam flow in turbines.

The high speed of the gas is obtained or explained by the combination of heat transfer and the friction to the flow. For a long pipe, the pressure difference reduces the density of the gas.For instance, in a perfect gas, the density is inverse of the pressure (it has to be kept in mind that the gas undergoes an isothermal process.). To maintain conservation of mass, the velocity increases inversely to the pressure. At critical point the velocity reaches the speed of sound at the exit and hence the flow will be choked.



Adiabatic (Q = 0), Frictional (Mathematically more difficult) (Short Insulated Pipes)

HOW TO DRAW A FANNO-LINE IN AN h-s DIAGRAM

A steady one-dimensional flow in a constant area duct with friction in the absence of
Work and heat transfer is known as “fanno flow”.

INTRODUCTION

Friction is present in all real flow passages. There are many practical flow situations where the effect of wall friction is small compared to the effect produced due to other driving potential like area, transfer of heat and addition of mass. In such situations, the result of analysis with assumption of frictionless flow does not make much deviation from the real situation. Nevertheless; there are many practical cases where the effect of friction cannot be neglected in the analysis in such cases the assumption of frictionless flow leads to unrealistic influence the flow. In high speed flow through pipe lines for long distances of power plants, gas turbines and air compressors, the effect of friction on working fluid is more than the effect of heat transfer ,it cannot be neglected An adiabatic flow with friction through a constant area duct is called fanno flow when shown in h-s diagram, curves ,obtained are fanno lines. Friction induces irreversibility resulting in entropy increase. The flow is adiabatic since no transfer of heat is assumed.

Applications

Fanno flow occurs in many practical engineering applications of such flow includes

1. Flow problems in aerospace propulsion system.

2. Transport of fluids in a chemical process plants.

3. Thermal and nuclear power plants.

4. Petrochemical and gas industries.

5. Various type of flow machineries.

6. Air conditioning systems.

7. High vacuum technology.

8. Transport of natural gas in long pipe lines.

10. Emptying of pressured container through a relatively short tube

11. Exhaust system of an internal combustion engine

12. Compressed air systems

When gases are transported through pipe over a long distances. It is also a practical importance when equipment handling gases are connected to high pressure reservoirs which may be located some distance away. Knowledge of this flow will allow us to determine the mass flow rate that can be candled, pressure drop etc…

In real flow, friction at the wall arises due to the viscosity of the fluid and this appears in the form of shear stress at the walls far in our discussion, we have assumed the fluid to be calorically perfect in viscid as well. Thus, strictly speaking, viscous effects cannot be accounted for in this formulation.However,in reality, viscous effects are confined to very thin region (boundary layer)near the walls. Effects such as viscous dissipation are also usually negligible.Hence, we can still assume the fluid to be inviscid and take the friction force exerted by the wall as an externally imposed force. The origin of this force is of significance to the analysis.

The following are the main assumptions employed for analyzing the frictional flow problem. in fanno flow

- One dimensional steady flow.
- Flow takes place in constant sectional area.
- There is no heat transfer or work exchange with the surroundings.
- The gas is perfect with constant specific heats.
- Body forces are negligible.

- Wall friction is a sole driving potential in the flow.
- There is no obstruction in the flow.
- There is no mass addition or rejection to or from the flow.

In thermodynamics coordinates, the fanno flow process can be described by a curve know as Fanno line and it's defined as the locus of the state which satisfies the continuity and energy and entropy equation for a frictional flow is known as "fanno line".

Fanno line or Fanno curve (Governing equation)

Flow in a constant area duct with friction and without heat transfers is described by a curve is known as Fanno line or Fanno curve

We know that

From continuity equation,

$m=\rho Ac$

$\frac{m}{A} = \rho c$

$G=mA=\rho c$

Where

G- Mass flow density.

c- Velocity of sound.

ρ - Density of fluid.

$G = \rho c$

$c = \frac{G}{\rho}$

$ho=h+12\times G2\rho 2$

$h=ho-12\times G2\rho 2$

Density ρ is a function of entropy and enthalpy.

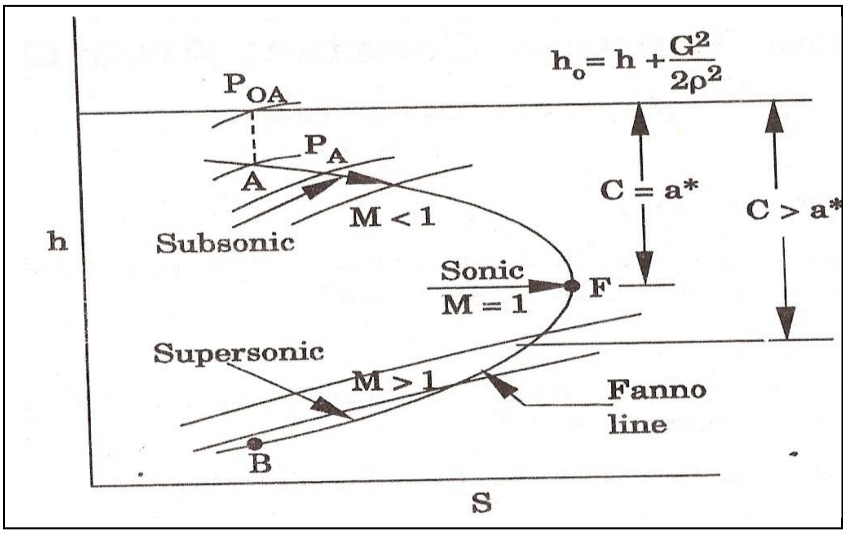
$\rho = f(s, h)$

Substitute the value for ρ in the equation for ‘h’

We get,

$h=ho-12*G22\{fs,h]2$

The above equation can be used to show a fanno-line in h-s diagram.



In the line

Point F is the sonic point

Point lying below are super sonic points

Points lying above are subsonic flow

Since entropy can only increase the processes that happen will always coverage to the sonic point F

The curve consists of two branches AF and FB. At point F the flow is sonic i,e, M=1

The flow A to F is subsonic ($M<1$) and B to F is Supersonic ($M>$)

In subsonic flow region (A to F), the effect of friction will increase the velocity and Mach number and to decrease the enthalpy and pressure of the gas.

In supersonic flow region (B to F), the effect of friction will decrease the velocity and Mach number and to increase the enthalpy and pressure of the gas.

We know by the second law o thermodynamics that for an adiabatic flow, the entropy may increase but cannot decrease. So the processes in the direction F to A and F to B are not possible because they lead to decrease in entropy.

Fanno curves are drawn for different vales of mass flow density (G).When G increases, the velocity increases and pressure decreases in the sub sonic region. When G increases, the pressure increases and velocity decreases in the super sonic region

Important features of Fanno curve

1.From the second law of thermodynamics, the entropy of the adiabatic flow increases but not decreases. Thus, the path of states along the Fanno curve must be toward the right.

2. In the subsonic region, the effects of friction will be to increase the velocity and Mach number and to decrease the enthalpy and pressure of the stream .

3.In the supersonic region, the effects of friction will be to decrease the velocity and Mach number and to increase the enthalpy and pressure of the stream.

4.A subsonic flow can never become supersonic, due to the limitation of second law of thermodynamics, but in can approach to sonic i.e,$M=1$.

5.A supersonic flow can never become subsonic, unless a discontinuity (shock)is present.

6. In the case of isentropic stagnation, pressure is reduced whether the flow is subsonic or supersonic.

Explain chocking in Fanno flow?

In a fanno flow, subsonic flow region, the effect of friction will increase the velocity and Mach number and to decrease the enthalpy and pressure of the gas. In supersonic flow region, the effect of friction will decrease the velocity and Mach number and to increase the enthalpy and pressure of the gas. In both cases entropy increases up to limiting state where the Mach number is one (M=1) and it is constant afterwards. At this point flow is said to be chocked flow.

Adiabatic Flow of a Compressible Fluid Through a Conduit

Flow through pipes in a typical plant where line lengths are short, or the pipe is well insulated can be considered adiabatic. A typical situation is a pipe into which gas enters at a given pressure and temperature and flows at a rate determined by the length and diameter of the pipe and downstream pressure. As the line gets longer friction losses increase and the following occurs:

a) Pressure decreases
b) Density decreases
c) Velocity increases
d) Enthalpy decreases
e) Entropy increases

The question is "will the velocity continue to increasing until it crosses the sonic barrier?" The answer is NO. The maximum velocity always occurs at the end of the pipe and continues to increase as the pressure drops until reaching Mach 1. The velocity cannot cross the sonic barrier in adiabatic flow through a conduit of constant cross section. If an effort is made to decrease downstream pressure further, the velocity, pressure, temperature and density remain constant at the end of the pipe corresponding to Mach 1 conditions. The excess pressure drop is dissipated by shock waves at the pipe exit due to sudden expansion. If the line length is increased to drop the pressure further the mass flux decreases, so that Mach 1 is maintained at the end of the pipe.

TABLE 12–4

The effects of friction on the properties of Fanno flow

Property	Subsonic	Supersonic
Velocity, V	Increase	Decrease
Mach number, Ma	Increase	Decrease
Stagnation temperature, T_0	Constant	Constant
Temperature, T	Decrease	Increase
Density, ρ	Decrease	Increase
Stagnation pressure, P_0	Decrease	Decrease
Pressure, P	Decrease	Increase
Entropy, s	Increase	Increase

What is the effect of friction in supersonic flow of the following parameters

a) velocity b) pressure c) temperature

Flow properties at M = M* = 1 are used as reference values for non-dimensionalizing various properties at any section of the duct.

a) <u>VELOCITY</u>

$\frac{dc}{c}$ = 12 M2(1+ Υ−12M2) dM^2

Integrating between M=1 and M= M,

c∗cdcc = 1MdM22M2(1+Υ−12M2)

ln$\frac{c}{c^*}$ = lnM [Υ−12(1+Υ−12M2)]$^{1/2}$

$\frac{c}{c^*}$ = M [Υ+12(1+Υ−12M2)]$^{1/2}$

Applying this equation for section 1 and 2 of the duct

$\frac{c1}{c2}$ = $M1M1$[1+Υ−12M121+Υ−12M22]1/2

b) <u>PRESSURE</u>

p∗pdpp = - −1 M1+Υ−1M22M2+Υ−1M2 dM^2

lnp p∗ = ln$\frac{1}{M}$ [Υ+12(1+Υ−12M2)]

$\frac{p}{p^*}$ = 1M [Υ+12(1+Υ−12M2)]1/2

Applying this equation for section 1 and 2 of the duct

$\frac{p2}{p1}$ = $\frac{M1}{M2}$ [1+Υ−12M121+Υ−12M22]1/2

c) <u>TEMPERATURE</u>

T∗TdTT = -1MΥ−12(1+Υ−12M2) dM^2

lnTT∗ = ln Υ+12(1+Υ−12M2)

$\frac{T}{T^*}$ = Υ+12(1+Υ−12M2)

Applying this equation for section 1 and 2

$\frac{T2}{T1}$ = **1+Υ−12M121+Υ−12M22**

Variation of flow properties

The flow properties (P,T,ρ,C) at $M=M^*=1$are used as reference values for non-dimensionalizing various properties at any section of the duct.

TEMPERATURE

Stagnation temperature –Mach number relation

At critical state

$M=1$

$T_0 = T_{0*}$

$T = T^*$

$$\frac{T_{0*}}{T*} = 1 + \frac{\gamma - 1}{2}$$

$$\boxed{\frac{T_{0*}}{T*} = \frac{\gamma + 1}{2}}$$

$$\Rightarrow \frac{T}{T*} = \frac{\frac{T_{0*}}{T*}}{\frac{T_0}{T}} = \frac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2} M^2}$$

For fanno flow $T0 = T0*$ *constant*

$$\Rightarrow \boxed{\frac{T}{T*} = \frac{\gamma + 1}{2\left(1 + \frac{\gamma - 1}{2} M^2\right)}}$$

Applying this equation for section 1 and 2

$$\Rightarrow \frac{T_2}{T_1} = \frac{\frac{T_{01}}{T_1}}{\frac{T_{02}}{T_2}} = \frac{1 + \frac{\gamma - 1}{2} M_1{}^2}{1 + \frac{\gamma - 1}{2} M_1{}^2}$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} M_1{}^2}{1 + \frac{\gamma - 1}{2} M_1{}^2}}$$

(FOR FANNO FLOW T ^{0} REMAINS CONSTANT)

Velocity

Mach number M

$M = \frac{c}{a}$, $c = M \times \sqrt{RT\gamma}$

At critical state

M=1

C = C*

$T = T^*$

$$c^* = \sqrt{R\gamma T^*}$$

$$\Rightarrow \frac{C}{C^*} = \frac{M \times \sqrt{R\gamma T}}{\sqrt{R\gamma T^*}}$$

$$= M\left[\frac{T}{T^*}\right]^{\frac{1}{2}}$$

$$= M\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}$$

$$\boxed{\frac{C}{C^*} = M\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}}$$

Applying this equation for section 1 and 2

$$\frac{C_2}{C_1} = \frac{M_2 a_2}{M_1 a_1}$$

$$= \frac{M_2}{M_1} \times \frac{\sqrt{\gamma R T_2}}{\sqrt{\gamma R T_1}}$$

$$= \frac{M_2}{M_1} \times \left[\frac{T}{T^*}\right]^{\frac{1}{2}}$$

$$= \frac{M_2}{M_1} \times \left[\frac{1 + \frac{\gamma - 1}{2} M_1{}^2}{1 + \frac{\gamma - 1}{2} M_1{}^2} \right]^{\frac{1}{2}}$$

$$\frac{C_2}{C_1} = \frac{M_2}{M_1} \times \left[\frac{1 + \frac{\gamma - 1}{2} M_1{}^2}{1 + \frac{\gamma - 1}{2} M_2{}^2} \right]$$

Density

$$\rho = \frac{1}{c}$$

$$\frac{\rho}{\rho^*} = \frac{1}{\frac{c}{c^*}}$$

$$= \frac{1}{M \left[\frac{\gamma + 1}{2\left(1 + \frac{\gamma - 1}{2} M^2\right)} \right]^{\frac{1}{2}}}$$

$$= \frac{1}{M} \left[\frac{2\left(1 + \frac{\gamma - 1}{2} M^2\right)}{\gamma + 1} \right]^{\frac{1}{2}}$$

$$= \frac{1}{M} \left[\frac{2 + (\gamma - 1) M^2}{\gamma + 1} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2 + (\gamma - 1) M^2}{(\gamma + 1) M^2} \right]^{\frac{1}{2}}$$

$$\frac{\rho^*}{\rho} = \left[\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right]^{\frac{1}{2}}$$ Applying this equation for section 1 and 2

$$\frac{\rho_2}{\rho_1} = \frac{1}{\frac{C_2}{C_1}}$$

$$= \frac{1}{\frac{M_2}{M_1}\left[\frac{\left(1+\frac{\gamma-1}{2}M_1{}^2\right)}{\left(1+\frac{\gamma-1}{2}M2^2\right)}\right]^{\frac{1}{2}}}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_2{}^2}{1+\frac{\gamma-1}{2}M_1{}^2}\right]^{\frac{1}{2}}}$$

PRESSURE

Pressure= ρRT

At critical state

M=1 Expressions for $\frac{T_o}{T*}, \frac{p_o}{p*}$ and $\frac{\rho_o}{\rho*}$

Considering the *section (where M = 1) and its stagnation section

We have

$M \Rightarrow 1$

$T \Rightarrow T*$

$p \Rightarrow p*$

$\rho \Rightarrow \rho*$

$P^* = \rho R T^*$

$$\frac{P}{P^*} = \frac{\rho R T}{\rho R T^*}$$

$$= \frac{\rho}{\rho^*} \times \frac{T}{T^*}$$

$$= \frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}} \times \left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]$$

$$\frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}} \times \left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}} \times \left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}$$

$$\boxed{\frac{P}{P^*} = \frac{1}{M}\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}}$$

Applying this equation for section 1 and 2

$$= \frac{P_2}{P_1} = \frac{\rho_2 R T_2}{\rho_1 R T_1}$$

$$= \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^{\,2}}{1+\frac{\gamma-1}{2}M_1^{\,2}}\right]^{\frac{1}{2}} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{1}{2}} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{1}{2}}$$

$$\boxed{\frac{P_2}{P_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right]^{\frac{1}{2}}}$$

STAGNATION PRESSURE

$$\frac{P_0}{P} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{P_0 = P\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}}$$

Considering the *section (where M = 1) and its stagnation section

We have

$p_0 = p_0^*$

$p = p^*$

$T_0 = T^*$

$T = T^*$

$$p_0^* = P^*\left(\frac{T_o^*}{T^*}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_0}{P_0^*} = \frac{P\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}}{P^*\left(\frac{T_o^*}{T^*}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{P_0}{P_0^*} = \frac{P}{P^*}\frac{\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{T_o^*}{T^*}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$= \frac{P}{P^*} \times \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{T^*}{T_o^*}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{P}{P^*} \times \left(\frac{T_o^*}{T}\right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{T^*}{T_0^*}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{P}{P^*} \times \left(\frac{T^*}{T} \right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{1}{M} \left[\frac{\gamma + 1}{2\left(1 + \frac{\gamma - 1}{2} M^2\right)} \right]^{-\frac{1}{2}} \times \left[\frac{2\left(1 + \frac{\gamma - 1}{2} M^2\right)}{\gamma + 1} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{1}{M} \left[\frac{2\left(1 + \frac{\gamma - 1}{2} M^2\right)}{\gamma + 1} \right]^{-\frac{1}{2}} \left[\frac{2\left(1 + \frac{\gamma - 1}{2} M^2\right)}{\gamma + 1} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{1}{M} \left[\frac{2\left(1 + \frac{\gamma - 1}{2} M^2\right)}{\gamma + 1} \right]^{-\frac{1}{2} + \frac{\gamma}{\gamma-1}}$$

$$= \frac{1}{M} \left[\frac{2\left(1 + \frac{\gamma - 1}{2} M^2\right)}{\gamma + 1} \right]^{\frac{-\gamma+1+2\gamma}{2(\gamma-1)}}$$

$$\boxed{\frac{P_0}{P_{0^*}} = \frac{1}{M} \left[\frac{2\left(1 + \frac{\gamma - 1}{2} M^2\right)}{\gamma + 1} \right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Applying this equation for section 1 and 2

$$\boxed{P_0 = P \left(\frac{T_o}{T} \right)^{\frac{\gamma}{\gamma-1}}}$$

$$P_{02} = P_2 \left(\frac{T_{o2}}{T_2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{02} = P_1 \left(\frac{T_{o2}}{T_2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_{01}} = \frac{P_2 \left(\frac{T_{o2}}{T_2} \right)^{\frac{\gamma}{\gamma-1}}}{P_1 \left(\frac{T_{o2}}{T_2} \right)^{\frac{\gamma}{\gamma-1}}}$$

$$= \frac{P_2}{P_1} \times \left(\frac{T_{o2}}{T_2} \right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{T_{o2}}{T_2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{P_2}{P_1} \times \left(\frac{T_1}{T_2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{M_1}{M_2} \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{1}{2}} \times \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{M_1}{M_2} \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{1}{2}} \times \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{-\gamma}{\gamma-1}}$$

$$= \frac{M_1}{M_2} \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{\gamma-1-2\gamma}{2(\gamma-1)}}$$

$$= \frac{M_1}{M_2} \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$\boxed{\frac{P_{O2}}{P_{O1}} = \frac{M_1}{M_2} \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{(\gamma+1)}{2(\gamma-1)}}}$$

IMPULSE FUNCTION

Impulse function $= PA\,(1+ \gamma M^2)$

Considering the *section (where M = 1) and its stagnation section

We have

$F = F^*$

$p = p^*$

$A = A^* = 1$

$M = 1$

$$F^* = P^* A^* (1+\gamma)$$

$$\frac{F}{F^*} = \frac{PA(1+\gamma M^2)}{P^* A^* (1+\gamma)}$$

$$= \frac{\dfrac{(1+\gamma M^2)}{(1+\gamma)}}{\dfrac{P^*}{P}}$$

$$= \frac{\dfrac{(1+\gamma M^2)}{(1+\gamma)}}{\dfrac{P^*}{P}}$$

$$= \frac{\dfrac{(1+\gamma M^2)}{(1+\gamma)}}{M\left[\dfrac{2\left(1+\dfrac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}}$$

$$= \frac{(1+\gamma M^2)}{M(1+\gamma)\left[\dfrac{2\left(1+\dfrac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}}$$

$$\boxed{\frac{F}{F^*} = \frac{(1+\gamma M^2)}{M(1+\gamma)\left[\dfrac{2\left(1+\dfrac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}}}$$

Applying this equation for section 1 and 2

We know that

$$F = PA\left(1+\gamma M^2\right)$$

$$F_1 = P_1 A_1\left(1+\gamma M_1{}^2\right)$$

$$\frac{F_2}{F_1} = \frac{P_2 A_2}{P_1 A_1} \times \frac{\left(1+\gamma M_2{}^2\right)}{\left(1+\gamma M_1{}^2\right)}$$

$$= \frac{P_2}{P_1} \times \frac{\left(1+\gamma M_2{}^2\right)}{\left(1+\gamma M_1{}^2\right)}$$

$$= \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1{}^2}{1+\frac{\gamma-1}{2}M_2{}^2}\right]^{\frac{1}{2}} \times \frac{\left(1+\gamma M_2{}^2\right)}{\left(1+\gamma M_1{}^2\right)}$$

$$\frac{F_2}{F_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1{}^2}{1+\frac{\gamma-1}{2}M_2{}^2}\right]^{\frac{1}{2}} \times \frac{\left(1+\gamma M_2{}^2\right)}{\left(1+\gamma M_1{}^2\right)}$$

ENTROPY

Changing entropy is given by

$$\frac{s-s^*}{R} = -\ln\left[\frac{p_0}{P_0{}^*}\right]$$

$$= -\ln\left[\frac{p_0}{P_0{}^*}\right] \times \frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}$$

Changing entropy for section 1 and 2

$$\frac{S_2 - S_1}{R} = -\ln\left[\frac{p_{01}}{P_{02}}\right]$$

$$\boxed{\frac{S_2 - S_1}{R} = \ln \frac{M_2}{M_1}\left[\frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{(\gamma+1)}{2(\gamma+1)}}}$$

Variation of Mach number with duct length

The duct length required for the flow to pass from a given initial mach number M_1 to a given final mach number m2can be obtained from the following expression.

Mean friction coefficient with respect to duct length is given by

$$\bar{f} = \frac{1}{L_{MAX}}\int_0^{L_{MAX}} fdx$$
$$fdx$$

from

$$4f\frac{dx}{D} = \frac{1-M^2}{\gamma M^2\left(1+\frac{\gamma-1}{2}\right)}\frac{dM^2}{M^2}$$

integrating limits are x=0 to x= L_{MAX}

M=M to M=1

$$\int_0^{L_{MAX}} 4f\frac{dx}{D} = \int_0^1 \frac{1-M^2}{\gamma M^2\left(1+\frac{\gamma-1}{2}\right)}\frac{dM^2}{M^2}$$

$$4f\frac{dx}{D} = \int_0^1 \frac{1-M^2}{\gamma\left(1+\frac{\gamma-1}{2}\right)}\frac{1}{M^4}dM$$

$$= 4f\frac{dx}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\frac{1}{M^4}\ln\frac{(\gamma+1)M^2}{2\left(1+\frac{\gamma-1}{2}M^2\right)}$$

The distance (L) between two section of duct where the ach numbers M1 &M2 are given by

$$\boxed{4f\frac{L_{Max}}{D} = \left[4f\frac{L_{Max}}{D}\right]_{M_1} - \left[4f\frac{L_{Max}}{D}\right]_{M_2}}$$

Problems

1. Air flows through a pipe of 300 mm diameter. At inlet temperature is 35^0c, pressure is 0.6 bar and stagnation pressure is 12 bar.At a location 2 m down stream, the static pressure is 0.89 bar.Estimate the average friction coefficient between two section.

Given date: D = 300 mm; $T_1 = 35^0$c ; Po1= 12 bar ; P_2 = 12 bar

$$\frac{P_1}{P_{O1}} = \frac{0.6}{12} = 0.05$$

From Fanno table

$$[M]_{\frac{P_1}{P_{O1}}} = 2.6$$

$$\left[4f\frac{L_{Max}}{D}\right]_{M=2.6_1} = 0.453$$

To find M_2

$$\frac{P_2}{P_1} = \frac{\left(\frac{P_2}{P^*}\right)_{M_2}}{\left(\frac{P_2}{P^*}\right)_{M_1=2.6}}$$

$$\frac{0.89}{0.62} = \frac{\left(\frac{P_2}{P^*}\right)_{M_2}}{0.275}$$

$$\left(\frac{P_2}{P^*}\right)_{M_2} = 0.275 \times \frac{0.89}{0.62} = 0.4079$$

$$[M_2]_{0.479} = 2$$

$$\left[4f\,\frac{L_{Max}}{D}\right]_{M=2} = 0.305$$

$$\boxed{4f\,\frac{L_{Max}}{D} = \left[4f\,\frac{L_{Max}}{D}\right]_{M1=2.6} - \left[4f\,\frac{L_{Max}}{D}\right]_{M_2=2}}$$

$$= \left[4f\,\frac{L_{Max}}{D}\right]_{M1=2.6} - \left[4f\,\frac{L_{Max}}{D}\right]_{M_2=2} = 0.453 - 0.305 = 0.148$$

$$\left[4f\,\frac{2}{D}\right]_{M=2} = 0.148$$

$$f = \frac{0.148 \times 3}{8} = 5.5 \times 10^{-3}$$

Question-2: - Air having an Mach number of 5 is decelerated in a 7.5 cm internal diameter pipe to Mach number 3.compute the length of the pipe which will cause this deceleration if f =0.005 and γ =1.4.

Given date: D= 0.045 m; $M_1 = 5$; $M_2 = 3$; f=0.005 ;

From fanno table $M_1 = 5, \gamma = 1.4$

$$\left[\frac{4fL_{Max}}{D}\right]_{M1=5} = 0.694$$

From fanno table $M_1 = 3, \gamma = 1.4$

$$\left[\frac{4fL_{Max}}{D}\right]_{M2=3} = 0.522$$

$$\boxed{\frac{4fL}{D} = \left[\frac{4fL_{Max}}{D}\right]_{M1} - \left[\frac{4fL_{Max}}{D}\right]_{M_2}}$$

$$= 0.694 - 0.522 = 0.172$$

$$L = \frac{0.175 \times D}{4f} = \frac{0.172 \times 0.075}{4 \times 0.005} = 0.645m$$

Length of the pipe L=0.645 m

Question-2: -Air flowing in an insulated constant area duct , experiences an increase in Mach number from 0.2 to 0.6 as a result of friction. The initial pressure and temperature are 1.4 bar and 21^{0}C . What are the final pressure and final velocity?

Given Data: M_1 =0.2 ; M_2 = 0.6 ; P_1 = 1.4 bar ; T_1 = 294 K

$$C_1 = M_1\sqrt{\gamma RT_1} = 68.739 \mathrm{m/sec}$$

From fanno table $M_1 = 0.2, \gamma = 1.4$

$$\left(\frac{P_1}{P^*}\right) = 5.455; \left(\frac{C_1}{C^*}\right) = 0.218$$

From fanno table $M_1 = 0.6, \gamma = 1.4$

$$\left(\frac{P_2}{P^*}\right) = 1.763; \left(\frac{C_2}{C^*}\right) = 0.635$$

$$\left(\frac{P_2}{P_1}\right) = \frac{\left(\frac{p_2}{p^*}\right)_{M2}}{\left(\frac{p_1}{p^*}\right)_{M1}} = \frac{1.763}{5.455}$$

$$P_2 = \frac{\left(\frac{p_2}{p^*}\right)_{M2}}{\left(\frac{p_1}{p^*}\right)_{M1}} \times P_1 = \frac{1.763}{5.455} \times 1.4 = 0.4524656 \mathrm{bar}$$

Similarly

C_2=0.6350.218×68.739=200.228m/sec

Question -3:- A circular duct passes 8.25 Kg / S of air at an exit Mach number of 0.5. The entry pressure and temperature are 345 KPa and 38^0C respectively and the co – efficient of friction 0.005. If the Mach number at entry is 0.15, determine,

(i) The diameter of the duct,

(ii) Length of the duct,

(iii) Pressure and temperature at exit and

(iv) Stagnation pressure loss.

Given Data: m = 8.25 Kg / S ; M_2 = 0.5 ; P_1 = 345 KPa ; T_1 = 311K ; f = 0.005 ;

M_1 = 0.15

(i) **Diameter of the pipe**

$$M = \rho_1 A_1 C_1$$

$$= \frac{P_1}{RT_1} A_1 M_1 \sqrt{\gamma R T_1}$$

$$m = \frac{P_1 A_1 \sqrt{\gamma} M_1}{\sqrt{RT_1}}$$

$$A_1 = \frac{m\sqrt{RT_1}}{\sqrt{\gamma} M_1 P_1} = \frac{8.25 \times \sqrt{287 \times 311}}{345 \times 10^3 \times 0.15\sqrt{1.4}}$$

$$= 0.040253m$$

$$A = \frac{\pi}{4} d^2$$

$$d = 0.226389$$

(ii) Length of the pipe

From Isentropic table M_1 = 0.15,γ =1.4

$$\frac{4fL}{D} = \left[\frac{4fL_{Max}}{D}\right]_{M1} - \left[\frac{4fL_{Max}}{D}\right]_{M_2}$$

=28.354-1.069

$$\frac{40.005 \times L}{0.226389} = 27.285$$
$$L = 308.85m$$

Pressure and temperature at the exit

$$\left(\frac{P_2}{P_1}\right) = \frac{\left(\frac{p_2}{p^*}\right)_{M2}}{\left(\frac{p_1}{p^*}\right)_{M1}} = \frac{2.138}{7.3193}$$

$$\left(\frac{P_2}{P_1}\right) = \frac{\left(\frac{p_2}{p^*}\right)_{M2}}{\left(\frac{p_1}{p^*}\right)_{M1}} = \frac{2.138}{7.3193}$$

= 100.773 KPa

$$T_2 = \frac{T_2 T_1^*}{T_2 T_1^*} \times T_1$$
$$= \frac{1.43}{1.1945} \times 311 = 297.59K$$

(iv)Stagnation presure loss

$$P_{02} = \frac{P_{02}}{P_{02} *} \times \frac{P_{01}}{P_{01} *} P_{01}$$
$$= \frac{1.34}{3.928} \times 350.609 = 231.009 \text{ кPA}$$

$$P_2 = \frac{\left(\frac{p_2}{p^*}\right)_{M2}}{\left(\frac{p_1}{p^*}\right)_{M1}} \times P_1 = \frac{2.629}{4.3615} \times 700$$
$$T_2 = \frac{1.161}{1.185} \times 333 = 326.2557K$$

$$C_2 = \frac{0.4415}{0.272} \times 0.25 \times \sqrt{1.4 \times 287 \times 333}$$
$$= 148.432 \text{m/sec}$$

$$\frac{T}{T*} = 1.185;$$

Question – 4 :- Air is flowing in an insulated duct with a Mach number of M1 = 0.25. At a section downstream the entropy is greater by amount 0.124 units, as a result of friction. What is the Mach number of this section? The static properties at inlet are 700KPa and 60^0C. Find velocity, temperature and pressure at exit. Fin d the properties at the critical section.

Given Data: M1 = 0.25; (S2 – S1) = 0.124 KJ/ Kg K; P1 = 700 KPa ; T1 = 60 + 273 = 333K

We know that

$$\frac{S_2 - S_1}{R} = -\ln\left[\frac{p_{01}}{P_{02}}\right]$$

$$\frac{S_2 - S_1}{R} = \ln\left[\frac{p_{01}}{P_{02}}\right]$$

$$\ln\left[\frac{p_{01}}{P_{02}}\right] = \frac{0.124}{0.287} = 0.4320557$$

$$\frac{p_{01}}{P_{02}} = 1.540421$$

From isentropic table $M_1 = 0.25, \gamma = 1.4$

$$\frac{p_{01}}{P_{02}} = 1.540421$$

$$\frac{T_1}{T_{01}} = 0.987 : \frac{p_1}{P_{01}} = 0.957$$
$$T_{01} = 337.386\text{k} : P_{01} = 731.452\text{kpa}$$

From fanno table $\gamma = 1.4$

$$\frac{p_{01}}{P_{02}} = 1.540421$$

$$\frac{\frac{p_{01}}{P_0 *}}{\frac{p_{02}}{P_0 *}} = 1.540421$$

$$\frac{p_{02}}{P_{01}} = \frac{2.4065}{1.540421} = 1.5622$$

From fanno table $\frac{p_0}{P_0 *}$ =1.5622, the corresponding Mach number M_2=0.41

b) Velocity ,pressure and Temperature at the exit section

$$T^* = \frac{T_1}{1.185} = \frac{333}{1.185}$$

$$T^* = 281.01266K$$

$$\frac{P}{P^*} = 4.3615$$

$$P^{*=}160.495Ka$$

$$C^* = \sqrt{\gamma RT} = 336.022m/\sec$$

Question-2: - Air flows through an adiabatic duct having a diameter of 20 cm. The inlet Mach number is 0.20 and Darcy's friction factor 0.01. If there a 10% loss in stagnation pressure, find the length of the pipe and Mach number. What is the percentage loss from inlet to a section at which the Mach number is 0.80?

Answer:-

<u>Case-1</u>

Given,

M_1=0.20

$4f=0.01$

$D=20cm=0.2m$

10% loss in stagnation pressure

From gas tables (Fanno flow table, $\gamma=1.4$ and $M_1=0.20$)

p01p01∗=2.964

$\frac{4fL_{max}}{D}=14.533$

We have,

p02 p02∗ = p02p01×p01p02∗

= p02p01×p01p01∗ [since $p_{02}*=p_{01}*$]

Here, $\frac{p_{02}}{p_{01}}=0.9$ (Since there is 10% loss in stagnation pressure)

Therefore,

p02p02∗=0.9 x 2.964

$\frac{p_{02}}{p_{02}*}=2.6676$

From gas tables (Fanno flow table, $\gamma=1.4$, p02p02∗=2.6676≈2.708)

$M_2=0.22$

$\frac{4fL_{max}}{D}=11.596$

4fLD=4fLmaxD M1−4fLmaxD M2

= 14.533-11.596

$\frac{4fL}{D}=2.937$

$L = \frac{2.937 \times D}{4f}$

= 2.937×0.20.01

L = <u>58.74 m</u>

Mach number at the section, M_2= <u>0.22</u>

Length of the pipe, (10% loss in stagnation pressure) =<u>58.74 m</u>

<u>Case-1</u>

Given,

$M_2=0.80$

From gas tables (Fanno flow table, $\gamma=1.4$ and $M_2=0.80$)

$\frac{p_{02}}{p_{02}^*} = 1.038$

We know that

$\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_{02}^*} \times \frac{p_{02}^*}{p_{01}}$

$= \frac{p_{02}}{p_{02}^*} \times \frac{p_{01}^*}{p_{01}}$ (Since $p_{02}^* = p_{01}^*$)

$= 1.038 \times \frac{1}{2.964}$ (Since $\frac{p_{01}}{p_{01}^*} = 2.964$)

$\frac{p_{02}}{p_{01}} = 0.3502$

Percentage of stagnation pressure loss

$\Delta p_0 = \frac{p_{01} - p_{02}}{p_{01}}$

$= 1 - \frac{p_{02}}{p_{01}}$

$= 1 - 0.3502$

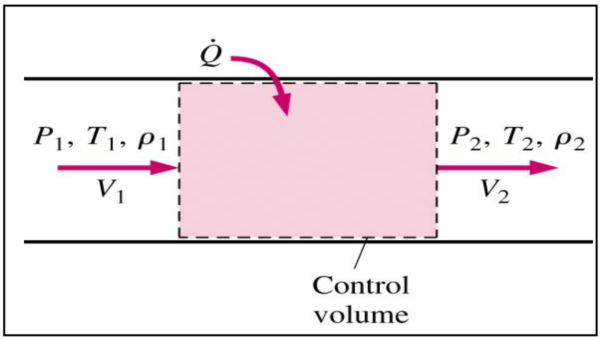
$= 0.6498$

$\Delta p_0 = \underline{\underline{64.98\%}}$

MODULE-4

Duct Flow with Heat Transfer and Negligible Friction

Flow in a constant area duct with heat transfer and without friction is known as Rayleigh flow . Many compressible flow problems encountered in practice involve chemical reactions such as combustion, nuclear reactions, evaporation, and condensation as well as heat gain or heat loss through the duct wall Such problems are difficult to analyze Essential features of such complex flows can be captured by a simple analysis method where generation/absorption is modeled as heat transfer through the wall at the same rate



In certain engineering processes, heat is added either by external sources across the system boundary by heat exchangers or internally by chemical reactions in a combustion chamber. Such process are not truely adiabatic, they are called adiabatic processes.

Applications

The combustion chambers inside turbojet engines usually have a constant area and the fuel mass addition is negligible. These properties make the Rayleigh flow model applicable for heat addition to the flow through combustion, assuming the heat addition does not result in dissociation of the air-fuel mixture. Producing a shock wave inside the combustion chamber of an engine due to thermal choking is very undesirable due to the decrease in mass flow rate and thrust. Therefore, the Rayleigh flow model is critical for an initial design of the duct geometry and combustion temperature for an engine.

The Rayleigh flow model is also used extensively with the Fanno flow model. These two models intersect at points on the enthalpy-entropy and Mach number-entropy diagrams, which is meaningful for many applications. However, the entropy values for each model are not equal at the sonic state. The change in entropy is 0 at M = 1 for each model, but the previous statement means the change in entropy from the same arbitrary point to the sonic point is different for the Fanno and Rayleigh flow models.

1. Combustion processes.

2. Regenerator,

3. Heat exchangers.

4. Inter coolers.

The following are the assumptions that are made for analyzing the such flow problem.

- One dimensional steady flow.
- Flow takes place in constant area duct.
- The frictional effects are negligible compared to heat transfer effects..
- The gas is perfect.
- Body forces are negligible.
- There is no external shaft work.
- There is no obstruction in the flow.
- There is no mass addition or rejection during the flow.
- The composition of the gas doesn't change appreciably during the flow.

Rayleigh line (or) curve

The frictionless flow of a perfect gas through a constant area duct in which heat transfer to or from the gas is the dominant factor bringing about changes in the flow is referred to as Rayleigh flow or diabetic flow. In thermodynamic coordinates, the Rayleigh flow process can be described by a curve known as Rayleigh line and is defined as the locus of quasi- static thermodynamic state points traced during the flow. The Rayleigh line satisfies the equation of state along with simple forms of continuity and momentum equation.

Governing Equations

In order to formulate the equation for the Rayleigh line, let us consider steady flow of a perfect gas through a constant area passage in which transfer of heat with the surroundings is the major factor responsible for changes in fluid properties. The simple form of continuity equation for steady one dimensional flow in a constant area duct is

$$m = \rho A c$$

$$\frac{m}{A} = \rho c$$

$$G = mA = \rho c$$

Where

G- Mass flow density.

c- Velocity of sound.

ρ - Density of fluid.

$$G = \rho c$$

$$c = \frac{G}{\rho}$$

Momentum equation is given by

$$p + \rho c^2 = cont.$$

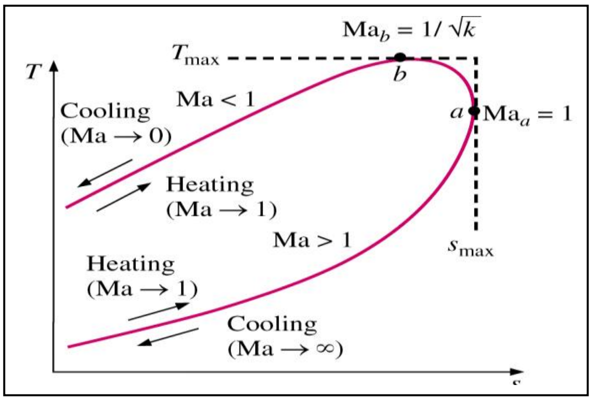
substitute

$$c = \frac{G}{\rho}$$

$$p + \frac{G^2}{\rho^2} = C$$

$p + G^2 v = c$ (Specific volume, v=1/ρ) -……………………………...1

Equation 1 may be used for representing Rayleigh line on the h- s diagram, as illustrated in fig shown in below. In general, most of the fluids in practical use have Rayleigh curves of the general form shown in fig.



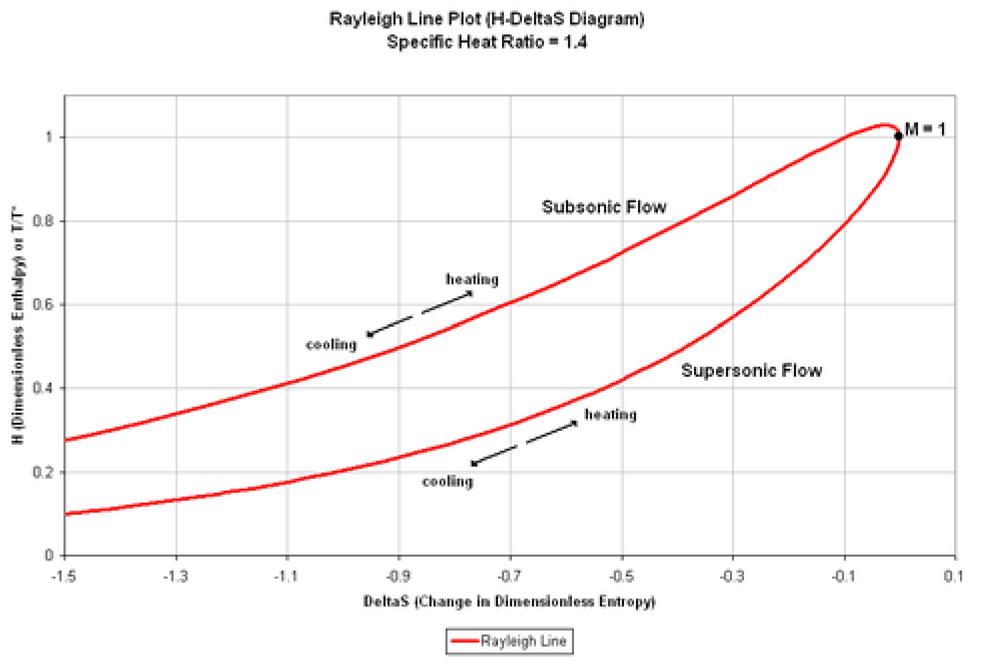
The portion of the Rayleigh curve above the point of maximum entropy usually represents subsonic flow (M<1) and the portion below the maximum entropy point represents supersonic flow (M>1).

An entropy increases due to heat addition and entropy decreases due to heat rejection. Therefore, the Mach number is increased by heating and decreased by cooling at subsonic speeds. On the other hand, the Mach number is decreased by heating and increased by cooling at supersonic speeds. Therefore, like friction, heat addition also tends to make the Mach number in the duct approach unity. Cooling causes the Mach number to change in the direction away from unity.

RAYLEIGH FLOW

Rayleigh flow refers to diabetic flow through a constant area duct where the effect of heat addition or rejection is considered. Compressibility effects often come into consideration, although the Rayleigh flow model certainly also applies to incompressible flow. For this model, the duct area remains constant and no mass is added within the duct. Therefore, unlike Fanno flow, the stagnation temperature is a variable. The heat addition causes a decrease in stagnation pressure which is known as the Rayleigh effect and is critical in the design of combustion systems. Heat addition will cause both supersonic and subsonic Mach numbers to approach Mach 1, resulting in choked flow. Conversely, heat rejection decreases a subsonic Mach number and increases a supersonic Mach number along the duct. It can be shown that for calorically perfect flows the maximum entropy occurs at M = 1. Rayleigh flow is named after John Strutt, 3rd Baron Rayleigh.

Theory



A Rayleigh Line is plotted on the dimensionless H-ΔS axis.

The Rayleigh flow model begins with a differential equation that relates the change in Mach number with the change in stagnation temperature, T_0. The differential equation is shown below.

$$\frac{dM^2}{M^2} = \frac{1+\gamma M^2}{1-M^2}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dT_0}{T_0}$$

Solving the differential equation leads to the relation shown below, where T_0* is the stagnation temperature at the throat location of the duct which is required for thermally choking the flow.

$$\frac{T_0}{T_0^*} = \frac{2(\gamma+1)M^2}{(1+\gamma M^2)^2}\left(1+\frac{\gamma-1}{2}M^2\right)$$

These values are significant in the design of combustion systems. For example, if a turbojet combustion chamber has a maximum temperature of T_0^* = 2000 K, T_0 and M at the entrance to the combustion chamber must be selected so thermal choking does not occur, which will limit the mass flow rate of air into the engine and decrease thrust.

For the Rayleigh flow model, the dimensionless change in entropy relation is shown below.

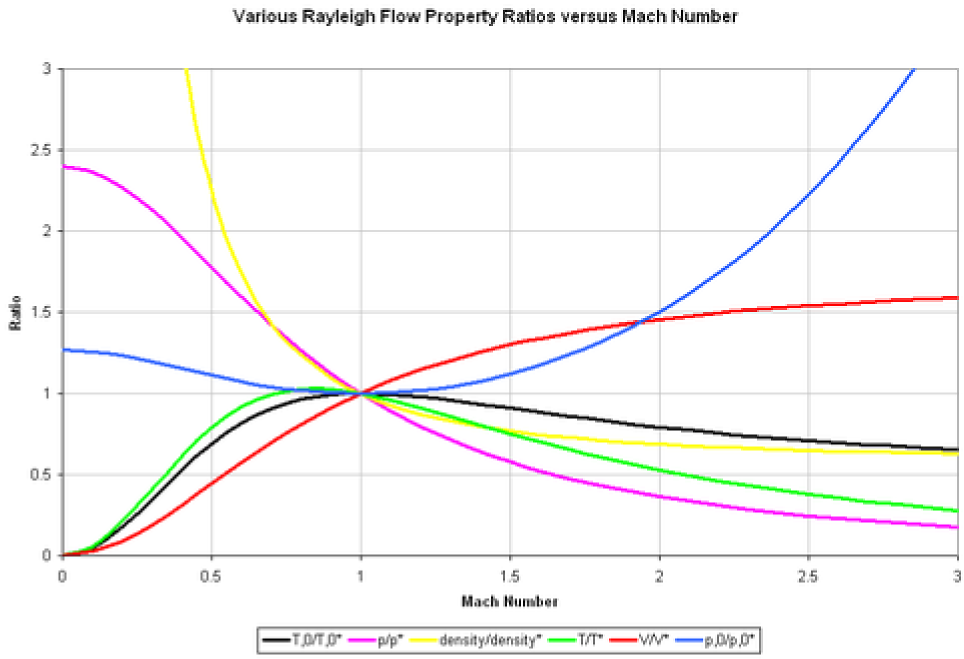
$$\Delta S = \frac{\Delta s}{c_p} = ln\left[M^2\left(\frac{\gamma+1}{1+\gamma M^2}\right)^{\frac{\gamma+1}{\gamma}}\right]$$

The above equation can be used to plot the Rayleigh line on a Mach number versus ΔS graph, but the dimensionless enthalpy, H, versus ΔS diagram is more often used. The dimensionless enthalpy equation is shown below with an equation relating the static temperature with its value at the choke location for a calorically perfect gas where the heat capacity at constant pressure, c_p, remains constant.

$$H = \frac{h}{h^*} = \frac{c_p T}{c_p T^*} = \frac{T}{T^*}$$
$$\frac{T}{T^*} = \left[\frac{(\gamma+1)M}{1+\gamma M^2}\right]^2$$

The above equation can be manipulated to solve for M as a function of H. However, due to the form of the T/T* equation, a complicated multi-root relation is formed for M = M(T/T*). Instead, M can be chosen as an independent variable where ΔS and H can be matched up in a chart as shown in Figure 1. Figure 1 shows that heating will increase an upstream, subsonic Mach number until M = 1.0 and the flow choke. Conversely, adding heat to a duct with an upstream, supersonic Mach number will cause the Mach number to decrease until the flow chokes. Cooling produces the opposite result for each of those two cases. The Rayleigh flow model reaches maximum entropy at M = 1.0 For subsonic flow, the maximum value of H occurs at M = 0.845. This indicates that cooling, instead of heating, causes the Mach number to move from 0.845 to 1.0 This is not necessarily correct as the stagnation temperature always increases to move the flow from a subsonic Mach number to M = 1, but from M = 0.845 to M = 1.0 the flow accelerates faster than heat is added to it. Therefore, this is a situation where heat is added but T/T* decreases in that region.

Additional Rayleigh Flow Relations



Various Rayleigh flow properties graphed as a function of Mach number.

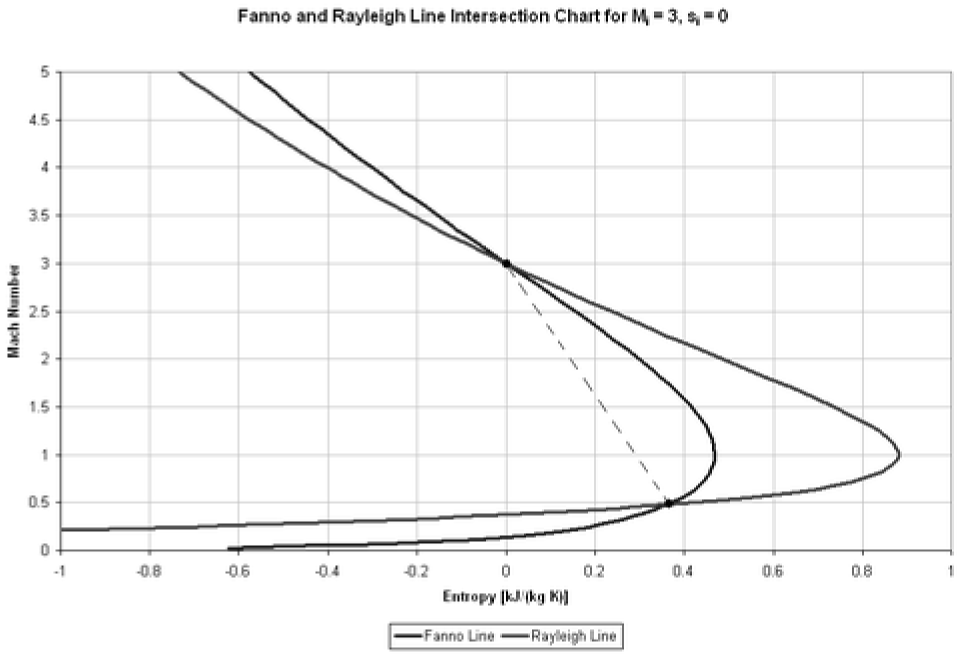
The area and mass flow rate are held constant for Rayleigh flow. Unlike Fanno flow, the Fanning friction factor, *f*, remains constant. These relations are shown below with the * symbol representing the throat location where choking can occur.

$$A = A^* = \text{constant}$$
$$\dot{m} = \dot{m}^* = \text{constant}$$

Differential equations can also be developed and solved to describe Rayleigh flow property ratios with respect to the values at the choking location. The ratios for the pressure, density, static temperature, velocity and stagnation pressure are shown below, respectively. They are represented graphically along with the stagnation temperature ratio equation from the previous section. A stagnation property contains a '0' subscript.

$$\frac{p}{p^*} = \frac{\gamma+1}{1+\gamma M^2}$$
$$\frac{\rho}{\rho^*} = \frac{1+\gamma M^2}{(\gamma+1)\,M^2}$$
$$\frac{T}{T^*} = \frac{(\gamma+1)^2\,M^2}{(1+\gamma M^2)^2}$$
$$\frac{V}{V^*} = \frac{(\gamma+1)\,M^2}{1+\gamma M^2}$$
$$\frac{p_0}{p_0^*} = \frac{\gamma+1}{1+\gamma M^2}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Applications



Fanno and Rayleigh Line Intersection Chart.

The Rayleigh flow model has many analytical uses, most notably involving aircraft engines. For instance, the combustion chambers inside turbojet engines usually have a constant area and the fuel mass addition is negligible. These properties make the Rayleigh flow model applicable for heat addition to the flow through combustion, assuming the heat addition does not result in dissociation of the air-fuel mixture. Producing a shock wave inside the combustion chamber of an engine due to thermal choking is very undesirable due to the decrease in mass flow rate and thrust. Therefore, the Rayleigh flow model is critical for an initial design of the duct geometry and combustion temperature for an engine.

The Rayleigh flow model is also used extensively with the Fanno flow model. These two models intersect at points on the enthalpy-entropy and Mach number-entropy diagrams, which is meaningful for many applications. However, the entropy values for each model are not equal at the sonic state. The change in entropy is 0 at M = 1 for each model, but the previous statement means the change in entropy from the same arbitrary point to the sonic point is different for the Fanno and Rayleigh flow models. If initial values of s_i and M_i are defined, a new equation for dimensionless entropy versus Mach number can be defined for each model. These equations are shown below for Fanno and Rayleigh flow, respectively.

$$\Delta S_F = \frac{s-s_i}{c_p} = ln\left[\left(\frac{M}{M_i}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{1+\frac{\gamma-1}{2}M_i^2}{1+\frac{\gamma-1}{2}M^2}\right)^{\frac{\gamma+1}{2\gamma}}\right]$$

$$\Delta S_R = \frac{s-s_i}{c_p} = ln\left[\left(\frac{M}{M_i}\right)^2\left(\frac{1+\gamma M_i^2}{1+\gamma M^2}\right)^{\frac{\gamma+1}{\gamma}}\right]$$

Figure shows the Rayleigh and Fanno lines intersecting with each other for initial conditions of $s_i = 0$ and $M_i = 3.0$ The intersection points are calculated by equating the new dimensionless entropy equations with each other, resulting in the relation below.

$$\left(1+\frac{\gamma-1}{2}M_i^2\right)\left[\frac{M_i^2}{(1+\gamma M_i^2)^2}\right] = \left(1+\frac{\gamma-1}{2}M^2\right)\left[\frac{M^2}{(1+\gamma M^2)^2}\right]$$

Interestingly, the intersection points occur at the given initial Mach number and its post-normal shock value. these values are M = 3.0 and 0.4752, which can be found the normal shock tables listed in most compressible flow textbooks. A given flow with a constant duct area can switch between the Rayleigh and Fanno models at these points.

Fundamental Equations

The following fundamental equations will be used to determine the variation of flow parameters in Rayleigh flows.

Continuity equation

We know that

Mass flow rate,

$$m = \rho_1 A_1 C_1 = \rho_2 A_2 C_2$$

For constant area duct $A_1 = A_2$

$$m = \rho_1 C_1 = \rho_2 C_2$$

$$\rho_1 C_1 = \rho_2 C_2$$

$$\frac{C_1}{C_2} = \frac{\rho_2}{\rho_1}$$

Where ;-

C_1 –Velocity of fluid at inlet-m/s

C_2 –Velocity of fluid at outlet-m/s

ρ_1 - Density of fluid at inlet-kg/m^3

ρ_2 - Density of fluid at out let-kg/m^3

Momentum equation

Momentum equation between state 1nad 2 is given by

$$p_1 A + mc_1 = p_2 A + mc_2$$

$$P_1 A - P_2 A = mc_2 - mc_1$$

$$\left(P_1 - P_2\right) A = m\left(c_2 - c_1\right)$$

Mass flow rate; $m=\rho Ac$

$$\left(P_1 - P_2\right) A = \rho AC\left(c_2 - c_1\right)$$

$$= \rho_2 AC^2{}_2 - \rho_1 AC^2{}_1$$

$$\left(P_1 - P_2\right) A = \rho_2 AC^2{}_2 - \rho_1 AC^2{}_1$$

$$\left(P_1 - P_2\right) = \rho_2 C^2{}_2 - \rho_1 C^2{}_1$$

$$\left[\rho = \frac{\mathrm{P}}{\mathrm{RT}}\right]$$

$$= \frac{\mathrm{P}_2}{\mathrm{RT}_2} \times M^2{}_2 \times \mathrm{a}_2{}^2 - \frac{\mathrm{P}_1}{\mathrm{RT}_1} \times M^2{}_1 \times \mathrm{a}_1{}^2$$

$$= \frac{\mathrm{P}_2 \times M^2{}_2 \times \sqrt{\gamma \mathrm{RT}_2}}{\mathrm{RT}_2} - \frac{\mathrm{P}_1 \times M^2{}_1 \times \sqrt{\gamma \mathrm{RT}_1}}{\mathrm{RT}_1}$$

$$\left(P_1 - P_2\right) = P_2 M^2{}_2 \gamma - P_1 M^2{}_1 \gamma$$

$$P_1 + P_1 M^2{}_1 \gamma = P_2 + P_2 M^2{}_2 \gamma$$

$$P_1\left[1 + M^2{}_1 \gamma\right] = P_2\left[1 + M^2{}_2 \gamma\right]$$

$$\boxed{\frac{P_2}{P_1} = \frac{\left[1 + M^2{}_1 \gamma\right]}{\left[1 + M^2{}_2 \gamma\right]}}$$

MACH NUMBER

The Mach number at the two states are

$$M_1 = \frac{C_1}{a_1}; M_2 = \frac{C_2}{a_2}$$

$$\frac{M_2}{M_1} = \frac{\frac{C_2}{a_2}}{\frac{C_1}{a_1}}$$

$$= \frac{C_2}{C_1} \times \frac{a_1}{a_2}$$

$$\frac{C_2}{C_1} \times \frac{\sqrt{\gamma R T_1}}{\sqrt{\gamma R T_2}}$$

$$\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left[\frac{T_1}{T_2}\right]^{\frac{1}{2}}$$

IMPULSE FUCTION

$$F = P\left[1 + M^2\gamma\right]$$

$$F_1 = P_1\left[1 + M_1{}^2\gamma\right]$$

$$F_2 = P_2\left[1 + M_2{}^2\gamma\right]$$

$$\frac{F_2}{F_1} = \frac{1 + M_2{}^2\gamma}{1 + M_1{}^2\gamma} \times \frac{P_2}{P_1}$$

$$\frac{P_2}{P_1} = \frac{1 + M_1{}^2\gamma}{1 + M_2{}^2\gamma}$$

$$\frac{F_2}{F_1} = \frac{P_1}{P_2} \times \frac{P_2}{P_1}$$

$$\frac{F_2}{F_1} = 1$$

STAGNATION PRESSUE

Stagnation pressure-Mach number relation is given by

$$\frac{P_0}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{01}}{P_1} = \left[1 + \frac{\gamma - 1}{2} M_1^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_2} = \left[1 + \frac{\gamma - 1}{2} M_2^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_2} = \left[1 + \frac{\gamma - 1}{2} M_2^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\frac{P_{02}}{P_2}}{\frac{P_{01}}{P_1}} = \frac{\left[1 + \frac{\gamma - 1}{2} M_2^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma - 1}{2} M_1^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{P_{02}}{P_2} \times \frac{P_1}{P_2} = \frac{\left[1 + \frac{\gamma - 1}{2} M_2^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma - 1}{2} M_1^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\boxed{\frac{P_{02}}{P_2} = \frac{P_2}{P_1} \times \frac{\left[1 + \frac{\gamma - 1}{2} M_2^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma - 1}{2} M_1^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}}}$$

$$\boxed{\frac{P_{02}}{P_2} = \frac{1 + M_1^{\ 2}}{1 + M_2^{\ 2}} \times \frac{\left[1 + \frac{\gamma - 1}{2} M_2^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma - 1}{2} M_1^{\ 2}\right]^{\frac{\gamma}{\gamma-1}}}}$$

STATIC TEMPERATURE

ROM EQN. $\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left[\frac{T_1}{T_2}\right]^2$

$$\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left[\frac{T_1}{T_2}\right]^2$$

WE KNOW THAT

$$\rho_1 = \frac{P_1}{RT_1} \qquad \rho_2 = \frac{P_2}{RT_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{\frac{P_2}{RT_2}}{\frac{P_1}{RT_1}}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \times \frac{T_1}{T_2}$$

$$\frac{T_1}{T_2} = \frac{\rho_2}{\rho_1} \times \frac{P_1}{P_2}$$

$$\frac{T_1}{T_2} = \frac{C_1}{C_2} \times \frac{P_1}{P_2}$$

$$\frac{C_1}{C_2} = \frac{P_2}{P_1} \times \frac{T_1}{T_2} \qquad \left[\frac{\rho_2}{\rho_1} = \frac{C_1}{C_2}\right]$$

SUBSTITUTE $\frac{C_1}{C_2}$

$$\frac{T_1}{T_2} = \left[\frac{\mathrm{M}_2}{\mathrm{M}_1} \times \frac{P_2}{P_1} \times \frac{\mathrm{T}_1}{\mathrm{T}_2}\right]^2$$

$$\frac{T_1}{T_2} = \left[\frac{\mathrm{M}_2}{\mathrm{M}_1} \times \frac{P_2}{P_1}\right]^2 \times \frac{\mathrm{T}_1^2}{\mathrm{T}_2^2}$$

$$\frac{T_1}{T_2} \times \frac{\mathrm{T}_1^{\,2}}{\mathrm{T}_2^{\,2}} = \left[\frac{\mathrm{M}_2}{\mathrm{M}_1} \times \frac{P_2}{P_1} \right]^2$$

$$\frac{T_2}{T_1} = \left[\frac{\mathrm{M}_2}{\mathrm{M}_1} \times \frac{P_2}{P_1} \right]^2$$

$$\frac{T_2}{T_1} = \left[\frac{\mathrm{M}_2}{\mathrm{M}_1} \times \frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}} \right] \qquad \left[\frac{P_2}{P_1} = \frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}} \right]$$

$$\frac{T_2}{T_1} = \frac{\mathrm{M}_2}{\mathrm{M}_1} \times \left[\frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}} \right]^2$$

STAGNATION TEMPERATURE

STAGNATION TEMPERATURE – ACH NUMBER RELATION IS GIVEN BY

$$\frac{\mathrm{T}_0}{T} = \left[1 + \frac{\gamma - 1}{2} M_2^{\,2} \right]$$

$$\frac{\mathrm{T}_{01}}{T_1} = \left[1 + \frac{\gamma - 1}{2} M_1^{\,2} \right]$$

$$\frac{\mathrm{T}_{02}}{T_2} = \left[1 + \frac{\gamma - 1}{2} M_2^{\,2} \right]$$

$$\frac{\dfrac{\mathrm{T}_{02}}{T_2}}{\dfrac{\mathrm{T}_{01}}{T_1}} = \frac{\left[1 + \dfrac{\gamma - 1}{2} M_2^{\,2} \right]}{\left[1 + \dfrac{\gamma - 1}{2} M_1^{\,2} \right]}$$

$$\frac{T_{02}}{T_{01}} = \frac{M_2^{\,2}}{M_1^{\,2}} \times \frac{\left(1+\gamma M_1^{\,2}\right)^{2}}{\left(1+\gamma M_2^{\,2}\right)^{2}} \times \frac{\left[1+\frac{\gamma-1}{2}M_2^{\,2}\right]}{\left[1+\frac{\gamma-1}{2}M_1^{\,2}\right]}$$

$$\left[\frac{T_2}{T_1} = \frac{M_2^{\,2}}{M_1^{\,2}} \times \frac{\left(1+\gamma M_1^{\,2}\right)^{2}}{\left(1+\gamma M_2^{\,2}\right)^{2}}\right]$$

$$\boxed{\frac{T_{02}}{T_{01}} = \frac{M_2^{\,2}}{M_1^{\,2}} \times \frac{\left(1+\gamma M_1^{\,2}\right)^{2}}{\left(1+\gamma M_2^{\,2}\right)^{2}} \times \frac{\left[1+\frac{\gamma-1}{2}M_2^{\,2}\right]}{\left[1+\frac{\gamma-1}{2}M_1^{\,2}\right]}}$$

CHANGE OF ENTROPY

$$S_2 - S_1 = C_P \ln\left[\frac{T_2}{T_1}\right] - C_P \ln\left[\frac{P_2}{P_1}\right]^{\frac{\gamma+1}{\gamma}}$$

$$S_2 - S_1 = C_P \ln\left[\frac{\left[\frac{M_2}{M_1} \times \frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}\right]^{2}}{\left[\frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}\right]^{\frac{\gamma+1}{\gamma}}}\right]$$

$$S_2 - S_1 = C_P \ln\left[\left(\frac{M_2^{\,2}}{M_1^{\,2}}\right) \times \left(\frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}\right)^{\left(\frac{\gamma+1}{\gamma}\right)}\right]$$

HEAT TRANSFER

WE HAVE

$$Q = mc_p\left(T_{02} - T_{01}\right)$$

$$Q = mc_p T_{01}\left(\frac{T_{02}}{T_{01}} - 1\right)$$

÷ BY c_p T₁

$$\frac{Q}{c_p T_1} = \frac{T_{01}}{T_1}\left(\frac{T_{02}}{T_{01}} - 1\right) = \quad = \left[1 + \frac{\gamma - 1}{2} M_1^2\right]\left[\frac{T_2}{T_1} \frac{\left[1 + \frac{\gamma - 1}{2} M_2^2\right]}{\left[1 + \frac{\gamma - 1}{2} M_1^2\right]}\right]$$

$$\frac{Q}{c_p T_1} = \frac{\left(M_2^2 - M_1^2\right)\left(2 - 2\gamma M_2^2 M_1^2 + [\gamma - 1]\left(M_2^2 + M_1^2\right)\right)}{2M_1^2\left(1 + \gamma \mathrm{M}_2^2\right)^2}$$

EXPRESSION FOR HEAT TRANSFER

WE HAVE

$$Q = mc_p\left(T_{02} - T_{01}\right)$$

THE CONDITION 1 IS FIXED BUT THE VALUE OF T_{02} ATTAINS ITS MAXIMUM WHEN

$T_{02} = T_0^*$

PROBLEMS

The condition of gas in a combustion chamber at entry are M1=0.28, T_{01}=380 K, P_{01}=4.9 bar. The heat supplied in the combustion chamber is 620 kJ/kg.Determine Mach number, pressure and temperature of the gas at exit and also determine the stagnation pressure loss during heating. Take γ = 1.3, c_p=1.22 kJ/Kg K.

Given,

$M_1 = 0.28$,

$T_{01} = 380$ K,

$P_{01} = 4.9$ bar $= 4.9 \times 10^5$ N/m²

Q = 620 kJ/kg = 620×10^3 J/kg

Take γ = 1.3, c_p=1.22 kJ/Kg K.= 1.22×10^3 J/kg K

To find

1. Mach number, pressure and temperature of the gas at exit,(M_2,P_2 and T_2)

2. Stagnation pressure loss (Δp_0)

Solution

Refer Isentropic flow table for γ=1.3 and M_1=0.28

$$\frac{T_2}{T_{01}} = 0.988 \qquad \text{[From gas table]}$$

$$\frac{P_2}{P_{02}} = 0.951$$

$P_1 = P_{01} \times 0.951$

$= 4.9 \times 10^5 \times 0.951$

$P_1 = 4.659 \times 10^5 \ N/m^2$

$T_1 = T_{01} \times 0.988$

$= 380 \times 0.988$

$T_1 = 375.44 \ K$

Refer Rayleigh flow table for γ =1.3 and M_1=0.28

$$\frac{P_1}{P^*_1} = 2.087$$

$$\frac{P_{01}}{P^*_{01}} = 1.198$$

$$\frac{T_1}{T^*_1} = 0.342$$

$$\frac{T_{01}}{T^*_{01}} = 0.300$$

$$P^* = \frac{P_1}{2.087} = \frac{4.659 \times 10^5}{2.087} = 2.23 \times 10^5 \ N/m^2$$

$$= 2.23 \times 10^5 \ N/m^2$$

$$P^*_1 = P^*_2$$

$$P^*_{01} = \frac{P_{01}}{1.198}$$

$$= \frac{4.9 \times 10^5}{1.198} = 4.09 \times 10^5$$

$$P^*_{01} = 4.09 \times 10^5 \, N/m^2 = P^*_{02}$$

$$T^*_1 = \frac{T_1}{0.342} = \frac{375.44}{0.342} = 1097.77K = T^*_2$$

$$T^*_{01} = \frac{T_{01}}{0.300} = \frac{380}{0.300} = 1266.6K = T^*_{02}$$

We know

$$Q = mc_p \, (T_{02}-T_{01})$$

For unit mass

$$Q=c_p \, (T_{02}-T_{01})$$

$$620 \times 10^3 = 1.22 \times 10^3 \, [T_{02}-380]$$

$$[T_{02}-380]=508.19$$

$$T_{02}=888.19 \text{ K}$$

$$\frac{T_{02}}{T^*_{02}} = \frac{888.19}{1266.6} = 0.701$$

Refer Rayleigh flow table for γ = 1.3 and M2=0.52

[From gas table]

$M_2=0.52$

$$\frac{P_2}{P^*_2} = 1.702$$

$$\frac{T_2}{T^*_2} = 0.783$$

$$\frac{P_{02}}{P^*_{02}} = 1.702$$

[**Note**: Mach no $M_1 < 1$,so $M_2 < 1$]

$P_2 = P^*_2 \times 1.702$

$= 2.23\times10^5\times1.702$

$P_2=3.79 \times10^5$ N/m^2

$T_2=T_2^* \times 0.783$

$=1097.77\times0.783$

$T_2=859.55$ K

$P_{02} = p_{02}^* \times 1.103$

$= 4.09\times10^5\times1.103$

$P_{02} = 4.511\times10^5$ N/m^2

Stagnation pressure loss

$\Delta p_0 = p_{01}-p_{02}$

$= 4.9\times10^5- 4.511\times10^5$

$\Delta p_0=0.389\times10^5$ N/m^2

Result,

1) $M_2=0.52$

$P_2=3.79 \times10^5$

$T_2=859.55$ K

2) $\Delta p_0=0.389\times10^5$ N/m^2

2.In a heat exchanger the static temperature of air is raised from 27°c to 177°c. The inlet pressure is 1.03 bar and the inlet Mach number is 0.07 neglecting the effect of wall friction, determine the final Mach number.

Given:

T_1 = 27°c+273 = 300k

T_2 = 177°c+273 = 450k

P_1 =1.03 bar=1.03×10^5 N/m^2

M_1=0.07

For air γ=1.4 and R=287 J/ KgK

To find:

Final Mach number, M2

Solution

Refer Rayleigh flow table for γ=1.4 and M_1=0.07

$$\frac{T_1}{T*_1} = 0.0285 \text{ [from gas tables]}$$

$$T*_1 = \frac{T_1}{0.0285} = \frac{300}{0.0285} = 10{,}526.3\text{K}$$

T_1*=10,526.3K=T_2*   [T_1*=T_2*]

We know that

$$\frac{T_2}{T*_2} = \frac{450}{10526} = 0.0427$$

Refer Rayleigh flow table for γ=1.4 and $\frac{T_2}{T*_2} == 0.0427$

M_2=0.088 [From gas tables]

RESULT

Final Mach number, M_2=0.088

3.In a certain heat exchanger, the stagnation temperature of air is raised from 93^{0}c and 426^{0}c.If the inlet Mach number is 0.3, determine the final Mach number and percentage drop in pressure.

Given: T_{01} = 93+273 =366K   :T_{02} = 426+273 = 699K: M_1 = 0.3

From Rayleigh Table M_1 = 2.131:   γ = 1.4

$$\frac{P}{P*} = 2.131$$

$$\frac{T}{T*} = 0.347 : T*_0 = 1054.755K$$

$$\frac{T_{02}}{T_0*} = \frac{699}{1054.755} = 0.6627$$

From Rayleigh Table corresponding to $\frac{T_0}{T_0*} \approx 0.661$,the Mach number M_2=0.48.

$$\frac{p_2}{p*} = 1.815$$

Percentage drop in pressure

$$= \left(\frac{\frac{P_1}{P*} - \frac{P_1}{P*}}{\frac{P_1}{P*}} \right) \times 100$$

$$= \left(\frac{2.131 - 1.8515}{2.131} \right) \times 100$$

$$ie \frac{P_1 - P_2}{P_1} = 14.828\%$$

RESULT

Final Mach number, M_2=0.088

Percentages drop in pressure. $= 14.828\%$

Q 1 Air flow through a constant area duct with inlet temperature of 20^0c and inlet mach number of 0.5. What is the possible exit stagnation temperature? It is desired to transfer heat such that at exit of the duct the stagnation temperature is 1180 K. For this condition what must be the limiting inlet mach number? Neglect friction.

Given:

T_1= 20+273=293 K

$M_1 = 0.5$

$T_{02} = 1180$ K

For air $\gamma = 1.4$ and R= 287 J/kg K

To find:

1. Exit stagnation temperature, T_{02}
2. Limiting inlet mach number, M_{L1}

Solution:

Case (i)

Refer isentropic flow table for $\gamma = 1.4$ and $M_1 = 0.5$

$$\frac{T_1}{T_{01}} = 0.952$$

$$T_{01} = \frac{293}{0.952} 307.77K$$

Refer Rayleigh flow table for $\gamma = 1.4$ and $M_2 = 1$

$$\frac{T_{02}}{T*_{02}} = 1$$

Refer Rayleigh flow table for $\gamma = 1.4$ and $M_2 = 0.5$

$$\frac{T_{01}}{T*_{01}} = 0.691$$

$$T_{01} = \frac{\frac{T_{02}}{T_{02}*}}{\frac{T_{01}}{T_{01}}*} \times T_1$$

$$= \frac{1}{0.691} \times 307.77$$

= 445.39 K

Case (ii)

Exit stagnation temperature, $T_{02} = 1180$ K

At exit M_2=1

Refer Rayleigh flow table for γ=1.4 and M_2= 1

$$\frac{T_{02}}{T*_{02}} = 1$$

$$\frac{T_{02}}{T*_{02}} = \frac{T_{02}}{T*_{01}} \times \frac{T_{01}}{T*_{01}}$$

$$1 = \frac{1180}{307.77} \times \frac{T_{01}}{T*_{01}} =$$

$$\frac{T_{01}}{T*_{01}} = 0.260$$

Refer Rayleigh flow table for $\frac{T_{01}}{T*_{01}} 0.260 \,\&\, \gamma = 1.4$

Limiting Mach number (M_{L1}) = 0.255

Q 2 Air enters a combustion chamber with certain Mach number .Sufficient heat is added to obtain a stagnation temperature ratio of 3 & a final Mach number of 0.8 .Determine the Mach number at entry & the percentage loss in static pressure. Take γ=1.4 & C_P=1.005KJ/KgK.

Given:

Stagnation temperature ratio, $\frac{T_{02}}{T_{01}} = 3$

Final Mach number ,M_2=0.8

γ =1.4 C_P=1.005 K J/K g K =1005 J / Kg K

To find:

1. Mach number at entry, M_1
2. Percentage loss in static pressure,ΔP

Solution:

Refer Rayleigh flow table for γ=1.4 & M_2=0.8

$$\frac{P_2}{P*_2} 1.266$$

$$\frac{T_{02}}{T*_{02}} = 0.964$$

We know that

$$\frac{\frac{T_{02}}{T*_{02}}}{\frac{T_{01}}{T*_{01}}} = 3$$

$$\frac{0.964}{\frac{T_{01}}{T*_{01}}} = 3$$

$$\frac{T_{01}}{T*_{01}} = \frac{0.964}{3} = 0.321$$

Refer Rayleigh flow table for $\frac{T_{01}}{T*_{01}} = \frac{0.964}{3} = 0.321 \, \& \, \gamma = 1.4$

M_1=0.28

$$\frac{P_1}{P*_1} = 2.163$$

[Note : Exit mach number, M_2<1.So, M_1<1]

Percentage loss in static pressure, ΔP= $e\frac{P_1 - P_2}{P_1}$ = $\left(\frac{2.163 - 1.266}{2.163}\right)100$

$= 41.47\%$

Result:

1. Mach number at entry, M_2=0.28

2.Percentage loss in static pressure, ΔP = 41.47%

Q. Air is heated in a frictionless duct from an initial static properties of P_1=110 kPa and T_1 = 300K. Calculate the amount of hit necessary to check the flow at exit of the duct when the inlet Mach number is (I) 2.2 and (2) 0.22.

Given

P_1= 110 kPa = 110 x 10^3 Pa = 110 x 10^3 $N/_m{}^2$

T_1 =300 K

Case (I)
M_1= 2.2
Case(2)
M_2=0.22
To find:
Heat transferred the case (1) and case (2)

Solution:

Case (1)

M_1=2.2
Refer Isentropic flow table for M_1=2.2 and γ=1.4

$$\frac{T1}{T01} = 0.508 \frac{P1}{P01} = 0.0935$$

$$T_{01} = \frac{T1}{0.508} = \frac{300}{0.508}$$

$$\boxed{T_{01}=590.55K}$$

$$P_{01}= \frac{P1}{0.0935}$$

$$= \frac{110 \times 1000}{0.0935}$$

$$\boxed{P_{01}=11.76 \times 10^5 \ N/m^2}$$

Refer Rayleigh flow table for γ=1.4 and M_1=2.2

$$\frac{T01}{T01*} = 0.756$$

$$T01* = \frac{T01}{0.756} = \frac{590.55}{0.756}$$

$$\boxed{T_{01}^{*}=781.15 \ K = T_{02}^{*}}$$

Exit state is not given. So, we are assuming exit state Mach number is one, ≫ M_2=1 and γ=1.4.

$$\frac{T02}{T02*} = 1$$

$T_{02}=T_{02*}$
$T_{02}=781.15$ K
Heat transfer, Q = m C_p $[T_{02}-T_{01}]$
For unit mass, Q = C_p $[T_{02}-T_{01}]$
= 1005[781.15-590.55]

$$\boxed{Q=191.55 \times 10^3 \text{ J/kg}}$$

Case (2)

$M_1=0.22$
Refer Isentropic flow table for $M_1=2.2$ and $\gamma=1.4$

$$\frac{T1}{T01} = 0.990$$

$$\frac{p1}{p01} = 0.967$$

$$\gg T_{01}=\frac{T1}{0.990} = \frac{300}{0.990}$$

$$\boxed{T_{01}=303.03K}$$

Refer Rayleigh flow table for $\gamma=1.4$ and $M_1=0.22$

$$\frac{T01}{T01*}=0.206$$

$$T_{01}^{*}=\frac{To1}{0.206} = \frac{303.03}{0.206}$$

$$\boxed{T_{01}^{*}=1471.01K=T_{02}^{*}}$$

Exit state is not given. So, we are assuming exit state Mach number is one.
$M_2=1$
Refer Rayleigh flow table for $M_2=1$ and $\gamma=1.4$.

$$\frac{T01}{T02*} = 1$$

$T_{02}=T_{02}^{*}$

$\boxed{T_{02}=1471.01\ K}$

Heat transfer, $Q= m\ C_p\ [T_{02}-T_{01}]$
For unit mass, $Q= C_p\ [T_{02}-T_{01}]$
$=1005\ [1471.01-303.03]$

$\boxed{Q=1173.8 \times 10^{3}\ J/kg}$

Result:

1. $Q= 191.55 \times 10^{3}$ j/kg
2. $Q= 1173.8 \times 10^{3}$ j/kg

Q.A gas (γ=1.3 and R = 0.46 KJ / Kg K) at a pressure of 70 Kpa and temperature of 295 K enters a combustion chamber at a velocity of 75 m / sec. The heat supplied in a combustion chamber is 1250 KJ / Kg .Determine the Mach number, pressure and temperature of gas at exit.

Given: γ = 1.3:   R = 0.46 KJ / Kg K

P1 = 70 Kpa : T1 =295 K

C1 = 75 m/sec: Q = 1250 KJ / Kg

$$C_p = \frac{\gamma}{\gamma - 1} = 1.999333\ \text{KJ / Kg K}$$

$$M_1 = \frac{C_1}{\sqrt{\gamma R T_1}}\ \frac{75}{\sqrt{1.3 \times 460 \times 295}} = 01.785 \approx 0.18$$

From isentropic table M_1 = 0.18 , γ=1.3.

$$\frac{T_1}{T_0} = 0.995 : \qquad \frac{P}{P_0} = 0.979$$

$$T_{01} = 296.4824K \qquad P_{01} = 71.5015\ K\ pa$$

From Rayleigh table M1 = 0.18 , γ=1.3.

$$\frac{P}{P*} = 2.207 \qquad \frac{P_0}{P*_0} = 1.23 \qquad \frac{T}{T*} = 0.138$$

$$T_0 * = \frac{T_{01}}{0.138} = 2148.4231K$$

We know that

Heat transfer

$$T_0 * = \frac{T_{01}}{0.138} = 2148.4231K$$

$$Q = C_P \ (T_{01} - T_{01})$$

$$T_{02} = \frac{Q}{C_P} + T_{01} = \frac{1250}{1.92333} + 296.4824$$

$$= 923.5727K$$

From Rayleigh table corresponding to M_2 = 0.35

$$\frac{P}{P*} = 1.9835 \qquad \frac{\mathrm{T}}{\mathrm{T}*} = 0.482$$

$$P_2 = \frac{P_2}{P*} \times \frac{\mathrm{P}*}{\mathrm{P}_1} \times \mathrm{P}_1$$

$$= \frac{1.9835}{2.207} \times 70 = 62.911\mathrm{K}\ pa$$

$$T_2 = \frac{0.4802}{0.158} \times 295 = 900K$$

Result:

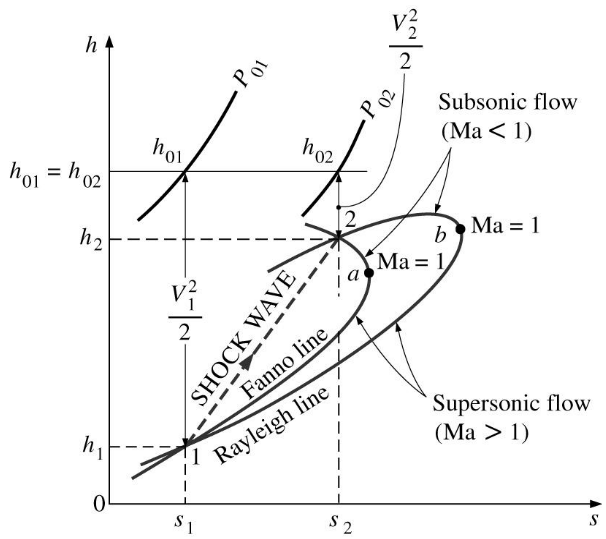
1. Mach number at the exit M_2 = 0.35

2. Pressure of the gas at the exit P_2 = 62.99111Kpa

3. Temperature of the gas at the exit T_2 = 900 K

Intersection of Fanno and a Rayleigh Line

Fanno and Rayleigh line, when plotted on h-s plane, for same mass velocity G, intersect at 1 and 2.as shown in fig. All states of Fanno line have same stagnation temperature or stagnation enthalpy, and all states of Rayleigh line have same stream thrust F / A. Therefore, 1 and 2 have identical values of G, h_0 and F / A. from 1 to 2 possible by a compression shock wave without violating Second Law Thermodynamics. A shock is a sudden compression which increases the pressure and entropy of the fluid but the velocity is decrease from supersonic to subsonic.



A change from states 2 to 1 from subsonic to supersonic flow is not possible in view Second Law Thermodynamics. (Entropy can not decrease in a flow process)

Q.1.The Mach number at the exit of a combustion chamber is 0.2. The ratio of stagnation temperatures at exit and entry is 3.74. If the pressure and temperature of the gas at exit is 2.5 bars and 1000^0c respectively determine (a) Mach number, pressure and temperature of the gas at entry, (b) the heat supplied per kg of the gas and (c) the maximum heat that can be supplied.

Take γ=1.3 and c_p=1.218 kJ/Kg K

Solution . T_2=1000+273=1273 K

From isentropic table for γ=1.3 at M_2=0.9

$$\frac{T_2}{T_{02}} = 0.892$$

T_{02}=1273/0.892=1427.13K

T_{01}=1427.13/3.74=381.58K

From Rayleigh tables for γ=1.3 at M2=0.9

P/P*=1.12, T/T*=1.017, T0/T_0* =0.991

$$\left(\frac{T0}{T0*}\right)_1 = \frac{T01}{T02}\frac{T02}{T0*} = \frac{0.991}{3.74} = 0.265$$

From Rayleigh tables for γ=1.3 at

$$\left(\frac{T0}{T0*}\right)_1 = 0.265$$

M_1=0.26, P/P*=2.114, T/T*=0.302

M_1=0.26 **Ans.**

$$P_1 = \frac{\left(\frac{p}{p}*\right)1}{\left(\frac{p}{p}*\right)2} \qquad p_2 = \frac{2.114}{1.12} \times 2.5$$

P_1=4.718 bar **Ans.**

$$T_1 = \frac{\left(\frac{T}{T}*\right)1}{\left(\frac{T}{T}*\right)2} \quad T_2 = \frac{0.302}{1.017} \times 1273$$

T_1=378 K **Ans.**

(b) $Q = c_p (T_{02} - T_{01})$

=1.218(1427.13 – 381.58)

Q=1273.48 kJ/kg **Ans.**

(c) $$Q_{max} = C_P\, T_1 \frac{(1-M_1^2)^2}{2(1+\gamma)M_1^2}$$

$$Q_{max} = \frac{1.218 \times 378(1-0.26)^2}{2 \times 2.3 \times 0.26^2}$$

Q_{max}= 1287.18kj/kg **Ans.**

Alternatively,

$$Q_{max} = C_P(T_0^*-T_{01})$$

$$T_0^*=\frac{T_{01}}{0.265}=\frac{T_{02}}{0.991}$$

$$T_0^*=\frac{381.58}{0.265}=\frac{1427.13}{0.991}=1440\ K$$

$$Q_{max}=1.218(1440-381.58)$$

$Q_{max}=1289$ kJ/kg **Ans.**

Q.2.The conditions of a gas in a combustor at entry are: $P_1=0.343$ bar, $T_1=310$ k, $C_1=60$ m/s

Determine the Mach number, pressure, temperature and velocity at the exit if the increase in stagnation enthalpy of the gas between entry and exit is 1172.5kJ/kg.

Take $C_p=1.005$kJ/kg K , $\gamma=1.4$.

Solution. $a_1=\sqrt{1.4\times 287\times 310}=353$ m/s

$$M_1=\frac{C_1}{a_1}=\frac{60}{353}=353\,m/s$$

$$T_{01}=\frac{T_1}{0.9943}=\frac{310}{0.9943}=311.78\ K$$

From isentropic tables for $\gamma=1.4$ $M_1=0.17$,

$$\frac{T_1}{T_{01}}=0.9943$$

$$T_{01}=\frac{T_1}{0.9943}=\frac{310}{0.9943}=311.78\ K$$

From Rayleigh tables for $\gamma=1.4$ at $M_1=0.17$

$$\frac{P}{P^*}=2.306 \qquad \frac{T}{T^*}=0.154 \qquad \frac{T_0}{T_0^*} \qquad \frac{C}{C^*}$$

$$T^*=\frac{T_{01}}{0129}=\frac{311.78}{0.129}=2416.89\ K$$

$$C_P\left(T_{02}-T_{01}\right)=\Delta \mathrm{h}_0$$

$$\left(T_{02}-T_{01}\right)=\frac{\Delta \mathrm{h}_0}{C_P}$$

$$=\frac{1172.5}{1.005}=1166.61K$$

$$T_{02}=1166.61+311.78=1478.45$$

$$\frac{T_{02}}{T*_0}=\frac{1478.45}{2416.89}=0.612$$

From Rayleigh tables for $\gamma=1.4$ at

$$\frac{T_{02}}{T*_0}=0.612$$

$M_2 = 0.45$ **Ans.**

$$\frac{P}{P*}=1.87,\ \frac{T}{T*}=0.7075,\ \frac{C}{C*}=0.378$$

$P_2 = \frac{1.87}{2.306}\times 0.343 = 0.278$ bar **Ans.**

$T_2 = \frac{0.7075}{0.154}\times 310 = 1424.18$ K **Ans.**

$C_2 = \frac{0.378}{0.0665}\times 60 = 341.05$ m/s **Ans.**

Alternatively,

$$a_2=\sqrt{\gamma R T_2}=\sqrt{1.4\times 287\times 1424.18}=757.26 \text{ m/s}$$

$C_2 = 0.45 \times 757.26 = 340.77$ m / s **Ans.**

12.The pressure, temperature and velocity of a gas in combustion chamber at entry are 0 .35 bar.300k and 55m/s. the increase in stagnation enthalpy of the gas between entry and exit is 1170 kJ/ Kg k. calculate the following:

Exit temperature, M2

Exit pressure,p_2

Exit temperature T_2

Exit velocity T_2

Cp =1.005 Kj / kgK, γ=1.4

Solution:

Mach no. at entry $$M_1 = \frac{C_1}{a} = \frac{C_1}{\sqrt{\gamma RT}} = \frac{55}{\sqrt{1.4x287x300}}$$

$$=0.152$$

From table, for γ=1.4and M_1=.16

$$\frac{T_1}{T_{01}} = 0.949$$

$$\frac{P_1}{P_{01}} = 0.982$$

$$T_{01} = \frac{T_1}{0.9949}$$

$$T_{01} = 301.54K$$

From Rayleigh's flow table for γ =1.4and M_1=0.158

$$\frac{P_1}{P^*} = 2.3$$

$$\frac{P_{01}}{P^*_{01}} = 1.24$$

$$\frac{T_1}{T^*} = 0.137$$

$$\frac{T_{01}}{T^*_{01}} = 0.115$$

$$\frac{C_1}{C_1*} = 0.59$$

$$P_1* = \frac{P_1}{2.31} = \frac{0.35\times10^5}{2.317} = 151\times10^5$$

$$P_1* = P_2* = 0.151\times10^5$$

$$T_{01}* = \frac{T_{01}}{0.115} = \frac{301.54}{0.115}$$

$$T_{01}* = T_{02}* \ 2622.08\,K$$

$$T* = \frac{T_1}{0.137} = \frac{30}{0.137} = 2189\,K$$

$$C_1* = \frac{C_1}{0.159} = \frac{5}{0.159}$$

$$C_1* = C*_2 = 932.2\,m/s$$

We know that

$$\Delta\text{h} = \text{C}_\text{P}\left(T_{02} - T_{01}\right)$$

1170 x10 3 = 1005 [T_{02} - 301.54]

1164.18 = T_{02} - 301.54

T_{02} = 1465.72 k

$$\frac{T_{02}}{T_{01}} = \frac{1465}{2622.08} = 0.56$$

From Rayleigh's flow table for γ=1.4and $\frac{T_{02}}{T_{01}} = \frac{1465}{2622.08}$=0.56 ,$M_2$ = 0.42

$$\frac{P_2}{P*_2} = 1.925$$

$$\frac{P_{02}}{P_{02}*} = 1.142$$

$$\frac{T_2}{T_2*} = 0.653$$

$$\frac{C_2}{C_2*} = 0.339$$

$$P_2 = P_2*\times 1.925$$

$$= 151\times 10^5 \times 1.925$$

$$= 0.2906 \times 10^5 \text{ N/m}^2$$

$T_2 = T_2* \text{ x}.653$

$= 2189.78 \text{ x}.653$

$= 1429.3\text{k}$

$C_2 = C_2* \text{ x}.339$

$= 932.2 \text{ x}.339$

$= 316.09$ m/s

14.The pressure temperature and Mach no. of air in combustion chamber are 4 bar,100°C and 0.2 respectively. The stagnation temperature of air in combustion chamber is increased 3 times the initial value. Calculate:

1. The mach no., pressure and temperature at exit.
2. Stagnation pressure
3. Heat supplied per Kg of air

Solution

Refer isentropic flow table for γ=1.4and m1=.2

$$\frac{T_1}{T_{01}} = 0.992$$

$$\frac{P_1}{P_{01}} = 0.973$$

$$T_{01} = \frac{T_1}{0.992}$$

$$= T_{01} = \frac{T_1}{0.992} = \frac{373}{0.992} = 376\,K$$

$T_{01} = 376K$

$$P_{01} = \frac{P_1}{0.973} = \frac{0.4 \times 10^5}{0.973}. = 4.1 \times 10^5\ N/m^2$$

$$P_{01} = \frac{P_1}{0.973}$$

$$P_{01} = 4.1x10^5\ N/m^2$$

From Rayleigh's flow table for $\gamma=1.4$and $M_1=.2$

$$\frac{P_1}{P^*} = 2.237$$

$$\frac{P_{01}}{P_{01}^*} = 1.235$$

$$\frac{T_1}{T^*_1} = 0.207$$

$$P_{1*} = \frac{P_1}{2.273} = 1.795\ x10^{\ 5}\,N/m2$$

$$P_{01*} = \frac{P_{01}}{1.235} = \frac{4.11\ x10^{\ 5}}{1.235} = 3.327x10^{\ 5} = N/m^2$$

$$T_1{*} = \frac{T_1}{0.207}$$

$$= \frac{373}{0.273}$$

$$= 1801.93 = T_2{*}$$

$$T^*{}_{01} = \frac{T_{01}}{0.174} = 2160.91 = T_{02}{*}$$

From given data we know that

$T_{02} = 3 \times T_{01}$

$= 3 \text{ x } 376$

$= 1128\text{ K}$

$$\frac{T_{02}}{T*_{02}} = \frac{1128}{2160.91} = 0.522$$

From Rayleigh's flow table for γ=1.4and $\frac{T_{02}}{T*_{02}}$=0.522

M_2=.4

$$\frac{P_2}{P_{2*}} = 1.961$$

$$\frac{P_{02}}{P_{02*}} = 1.961$$

$$\frac{T_2}{T_2*} = 0.615$$

$$P_2 = P*_2 \times 1.196$$
$$= 1.759 \times 10^5 \times 1.196$$
$$= \quad 3.44 \text{ x}10^{5} \text{N/m}^2$$

T_2=T_2* x.615

=1801.18K

P_{02}=3.849 x10 ^{5}N/m^2

Stagnation pressure loss

ΔP_0=P_{01}-P_{02}

=4.11 x10 ^{5}-3.84 x10 ^{5}

=.261 x10 ^{5}N/m2

Heat supplied Q=C_P(T_{02}-T_{01})

=1005(1128-376)

=755.7 x10 ^{3}J/kg

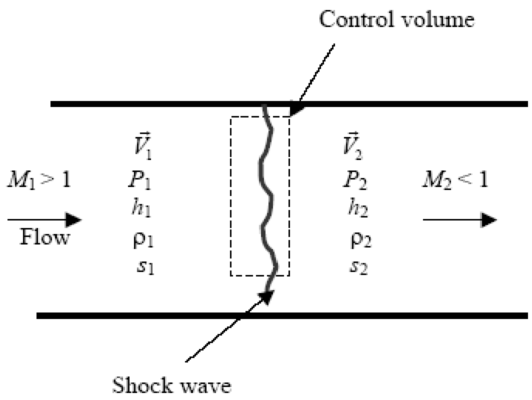
Normal Shocks

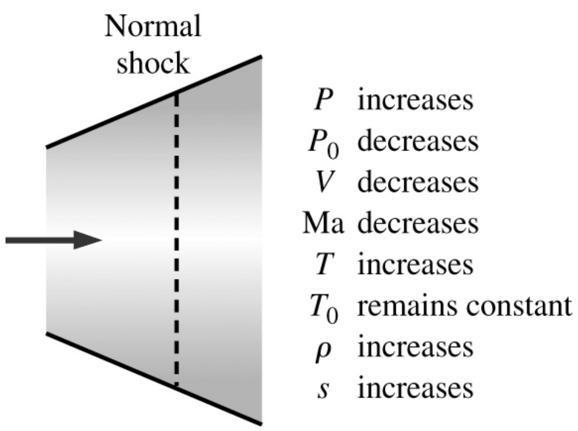
When there is a relative motion between a body and fluid, the disturbance is created if the disturbance is of an infinitely small amplitude, that disturbance is transmitted through the fluid with the speed of sound. If the disturbance is finite shock waves are created.

Shock Waves and Expansion Waves Normal Shocks

Shocks which occur in a plane normal to the direction of flow are called **normal shock waves.** Flow process through the shock wave is highly irreversible and *cannot* be approximated as being isentropic. Develop relationships for flow properties before and after the shock using conservation of mass, momentum, and energy

In some range of back pressure, the fluid that achieved a sonic velocity at the throat of a converging-diverging nozzle and is accelerating to supersonic velocities in the diverging section experiences a *normal shock*. The normal shock causes a sudden rise in pressure and temperature and a sudden drop in velocity to subsonic levels. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The properties of an ideal gas with constant specific heats before (subscript 1) and after (subscript 2) a shock are related by



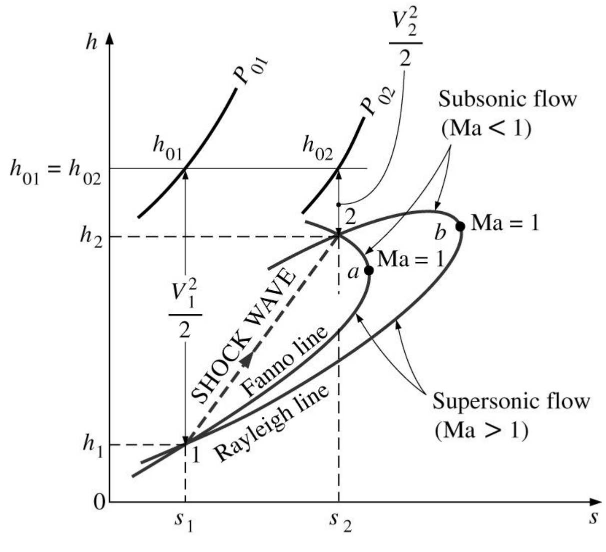


Assumptions

1. Steady flow and one dimensional
2. dA = 0, because shock thickness is small
3. Negligible friction at duct walls since shock is very thin
4. Zero body force in the flow direction
5. Adiabatic flow (since area is small)
6. No shaft work
7. Potential energy neglected

Intersection of Fanno and a Rayleigh Line

Fanno and Rayleigh line, when plotted on h-s plane, for same mass velocity G, intersect at 1 and 2.as shown in fig. All states of Fanno line have same stagnation temperature or stagnation enthalpy, and all states of Rayleigh line have same stream thrust F / A. Therefore, 1 and 2 have identical values of G, h_0 and F / A. from 1 to 2 possible by a compression shock wave without violating Second Law Thermodynamics. A shock is a sudden compression which increases the pressure and entropy of the fluid but the velocity is decrease from supersonic to subsonic.



A change from states 2 to 1 from subsonic to supersonic flow is not possible in view Second Law Thermodynamics. (Entropy can not decrease in a flow process)

Governing Equations

(i) Continuity

$$\dot{m}_x = \dot{m}_y$$

$$(\rho AV)_x = (\rho AV)_y$$

$\rho_x V_x = \rho_y V_y$ (Shock thickness being small $A_x = A_y$)

$G_x = G_y$ (Mass velocity)

Mass velocity G remains constant across the shock.

(ii) Energy equation

SFEE: $\dot{q} - \dot{\omega}_{sh} = h_2 - h_1 + \dfrac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$

$$h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2}$$

Across the shock

$$h_y + \frac{V_y^2}{2} = h_x + \frac{V_x^2}{2}$$

$$h_{ox} = h_{oy}$$

$$T_{ox} = T_{oy}$$

T_o remains constant across the shock.

(iii) Momentum

Newton's second law

$$\Sigma F_{xx} = \frac{\partial}{\partial t}(\dot{m}V_{xx})_{cv} + (\dot{m}V_{xx})_{out} - (\dot{m}V_{xx})_{in}$$

$$P_x A - P_y A = 0 + (\dot{m}V_{xx})_{out} - (\dot{m}V_{xx})_{in}$$

$$= \dot{m}(V_y - V_x) = \rho AV(V_y - V_x) = \rho_y AV_y^2 - \rho_x AV_x^2$$

Momentum gives

$$P_xA - P_yA = \rho_y A V_y^2 - \rho_x A V_x^2$$

$$P_xA + \rho_x A V_x^2 = P_yA + \rho_y A V_y^2$$

$$\therefore F_x = F_y$$

Impulse function remains constant across the shock.

<u>Property relations across the shock.</u>

(1) $\frac{T_y}{T_x}$

<u>Energy</u>

$$h_{ox} = h_{oy}$$

$$T_{ox} = T_{oy} \qquad (1)$$

For the isentropic x – ox

$$\frac{T_{ox}}{T_x} = 1 + \frac{K-1}{2} M_x^2$$

$$T_{ox} = T_x \left[1 + \frac{K-1}{2} M_x^2\right] \qquad \text{.......... (2)}$$

Similarly

$$T_{oy} = T_y \left[1 + \frac{K-1}{2} M_y^2\right] \qquad \text{............ (3)}$$

Combining (1), (2) and (3)

$$T_x \left[1 + \frac{K-1}{2} M_x^2\right] = T_y \left[1 + \frac{K-1}{2} M_y^2\right]$$

$$\frac{T_y}{T_x} = \left[\frac{1+\frac{K-1}{2}M_x^2}{1+\frac{K-1}{2}M_y^2}\right]$$

(ii) $\frac{P_y}{P_x}$

Momentum

$F_x = F_y$

$$P_xA + \rho_xAV_x^2 = P_yA + \rho_yAV_y^2$$

$$P_x\left[1+\frac{\rho_xV_x^2}{P_x}\right] = P_y\left[1+\frac{\rho_yV_y^2}{P_y}\right]$$

$$P_x\left[1+KM_x^2\right] = P_y\left[1+KM_y^2\right]$$

$$\frac{P_y}{P_x} = \left[\frac{1+KM_x^2}{1+KM_y^2}\right]$$

(iii) $\frac{\rho_y}{\rho_x}$

Equation of state $P = \rho RT$

$$P_x = \rho_xRT_x$$

$$P_y = \rho_yRT_y$$

$$\frac{\rho_x}{\rho_y} = \frac{\left(\frac{P_x}{RT_x}\right)}{\left(\frac{P_y}{RT_y}\right)} = \frac{P_xT_y}{T_xP_y}$$

$$\frac{\rho_x}{\rho_y} = \frac{1+KM_y^2}{1+KM_x^2}\left[\frac{1+\frac{K-1}{2}M_x^2}{1+\frac{K-1}{2}M_y^2}\right]$$

(iv) $\dfrac{V_y}{V_x}$

Continuity equation

$$\rho_x V_x = \rho_y V_y$$

$$\frac{V_y}{V_x} = \frac{\rho_x}{\rho_y}$$

Equation of state

$P = \rho RT$

$$P_x = \rho_x R T_x$$

$$P_y = \rho_y R T_y$$

$$\frac{V_y}{V_x} = \left[\frac{1+KM_y^2}{1+KM_x^2}\right]\left[\frac{1+\frac{K-1}{2}M_x^2}{1+\frac{K-1}{2}M_y^2}\right]$$

(v) $\dfrac{P_{oy}}{P_{ox}}$

For the isentropic x – ox

$$\frac{P_{ox}}{P_x} = \left(1+\frac{K-1}{2}M_x^2\right)^{\frac{K}{K-1}}$$

For the isentropic (y – oy)

$$\frac{P_{oy}}{P_y} = \left(1+\frac{K-1}{2}M_y^2\right)^{\frac{K}{K-1}}$$

$$P_{oy} = P_y\left(1+\frac{K-1}{2}M_y^2\right)^{\frac{K}{K-1}}$$

$$\frac{P_{oy}}{P_{ox}}=\frac{P_y}{P_x}\left[\frac{1+\frac{K-1}{2}M_y^2}{1+\frac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}$$

But $\frac{P_y}{P_x}=\frac{1+KM_x^2}{1+KM_y^2}$

$$\therefore\ \frac{P_{oy}}{P_{ox}}=\left[\frac{1+KM_x^2}{1+KM_y^2}\right]\left[\frac{1+\frac{K-1}{2}M_y^2}{1+\frac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}$$

(vi) Entropy change (ΔS)

$$\Delta S=S_y-S_x$$

$$=C_p\ln\frac{T_y}{T_x}-R\ \ln\frac{P_y}{P_x}=C_p\ln\ \frac{T_{oy}}{T_{ox}}-R\ \ln\ \frac{P_{oy}}{P_{ox}}=-R\ln\ \frac{P_{oy}}{P_{ox}}$$

$$\therefore\ S_y-S_x=-R\ \ln\left\{\left[\frac{1+KM_x^2}{1+KM_y^2}\right]\left[\frac{1+\frac{K-1}{2}M_y^2}{1+\frac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}\right\}\ \left[\because\ T_{oy}-T_{ox}\right]$$

(vii) Relation between M_x^2 and M_y^2

Prove that $M_y^2=\left[\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}\right]$

We have

(1) $\frac{T_y}{T_x}=\left[\frac{1+\frac{K-1}{2}M_x^2}{1+\frac{K-1}{2}M_y^2}\right]$

$$(2)\ \frac{P_y}{P_x} = \left[\frac{1+KM_x^2}{1+KM_y^2}\right]$$

$$(3)\ \frac{P_x M_x}{\sqrt{T_x}} = \frac{P_y M_y}{\sqrt{T_y}}$$

$$\text{Equation (3)} \Rightarrow \frac{P_y^2}{P_x^2} \cdot \frac{M_y^2}{M_x^2} = \frac{T_y}{T_x}$$

$$\left[\frac{1+KM_x^2}{1+KM_y^2}\right]^2 \left(\frac{M_y}{M_x}\right)^2 = \frac{1+\frac{K-1}{2}M_x^2}{1+\frac{K-1}{2}M_y^2}$$

$$\left(1+KM_x^2\right)^2 M_y^2 \left(1+\frac{K-1}{2}M_y^2\right) = \left(1+KM_y^2\right)^2 M_x^2 \left(1+\frac{K-1}{2}M_x^2\right)$$

$$\text{L.H.S.} = \left(1+2KM_x^2 + K^2M_x^4\right)M_y^2\left(1+\frac{K-1}{2}M_y^2\right)$$

$$= M_y^2\left(1+2KM_x^2+K^2M_x^4+\frac{K-1}{2}M_y^2+K\left(K-1\right)M_x^2M_y^2+\frac{K^2\left(K-1\right)}{2}M_x^4M_y^2\right)$$

$$= M_y^2+2KM_x^2M_y^2+K^2M_x^4M_y^2+\frac{K-1}{2}M_y^4+K\left(K-1\right)M_x^2M_y^4+\frac{K^2\left(K-1\right)}{2}M_x^4M_y^4$$

Similarly R.H.S.

$$= M_x^2+2K_x^2M_y^2+K^2M_y^2M_x^2+\frac{K-1}{2}M_x^4+K\left(K-1\right)M_y^2M_x^4+\frac{K^2\left(K-1\right)}{2}M_y^4M_x^4$$

$$=\left(M_y^2-M_x^2\right)+K^2\left(M_x^4M_y^2-M_y^4M_x^2\right)+\frac{K-1}{2}\left(M_y^4-M_x^4\right)+K\left(K-1\right)\left(M_x^2M_y^4-M_y^2M_x^4\right)$$

$$=\left(M_y^2-M_x^2\right)\left[1+\frac{K-1}{2}\left(M_y^2+M_x^2\right)-KM_x^2M_y^2\right]=0$$

Since $M_y \neq M_x$

So $\left(M_y^2-M_x^2\right)$ cannot be equal to zero

Hence $1+\frac{K-1}{2}\left(M_y^2+M_x^2\right)=KM_x^2M_y^2$

$$\frac{K-1}{2}\left(M_y^2+M_x^2\right)=KM_x^2M_y^2-1$$

$$\left(M_y^2+M_x^2\right)=\frac{2}{K-1}\left(KM_x^2M_y^2-1\right)$$

$$M_y^2=\frac{2K}{K-1}M_x^2M_y^2-\frac{2}{K-1}-M_x^2$$

$$M_y^2\left[1-\frac{2K}{K-1}M_x^2\right]=-\frac{2}{K-1}-M_x^2$$

$$M_y^2=\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}$$

Property relations in terms of incident Mach Number M_x.

(1) Prove $\frac{P_y}{P_x}=\frac{2K}{K-1}M_x^2-\frac{K-1}{K+1}$

Momentum $F_x = F_y$

$$P_x\left(1+KM_x^2\right)=P_y\left(1+KM_y^2\right)$$

$$\frac{P_y}{P_x}=\frac{1+KM_x^2}{1+KM_y^2}$$

but $M_y^2=\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}$

$$\frac{P_y}{P_x}=\frac{1+KM_x^2}{1+K\left[\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}\right]}=\frac{\left(1+KM_x^2\right)\left(\frac{2K}{K-1}M_x^2-1\right)}{\left(\frac{2K}{K-1}M_x^2-1\right)+K\left[\frac{2}{K-1}+M_x^2\right]}\qquad(1)$$

$$\text{Dinominator } D_r = \left(\frac{2K}{K-1}M_x^2 - 1\right) + K\left[\frac{2}{K-1} + M_x^2\right]$$

$$= M_x^2\left[\frac{2K}{K-1} + K\right] + \frac{2K}{K-1} - 1 \quad = KM_x^2\left[\frac{K+1}{K-1}\right] + \frac{K+1}{K-1}$$

$$= \frac{K+1}{K-1}\left(1 + KM_x^2\right) \text{ substitute in equation (1).}$$

$$\frac{P_y}{P_x} = \frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}$$

(2) Prove that $\dfrac{T_y}{T_x} = \dfrac{\left(1 + \frac{K-1}{2}M_x^2\right)\left(\frac{2K}{K-1}M_x^2 - 1\right)}{\frac{(K+1)^2}{2(K-1)}M_x^2}$

Energy

$$h_{ox} = h_{oy}$$

$$T_{ox} = T_{oy}$$

$$\frac{T_y}{T_x} = \left[\frac{1 + \frac{K-1}{2}M_x^2}{1 + \frac{K-1}{2}M_y^2}\right]$$

$$M_y^2 = \frac{\frac{2}{K-1} + M_x^2}{\frac{2K}{K-1}M_x^2 - 1}$$

$$\frac{T_y}{T_x} = \frac{1 + \frac{K-1}{2}M_x^2}{1 + \frac{K-1}{2}\left[\frac{\frac{2}{K-1} + M_x^2}{\frac{2K}{K-1}M_x^2 - 1}\right]} = \frac{1 + \frac{K-1}{2}M_x^2\left(\frac{2K}{K-1}M_x^2 - 1\right)}{\frac{2K}{K-1}M_x^2 - 1 + \frac{K-1}{2}\left(\frac{2}{K-1} + M_x^2\right)}$$

$$\text{Denominator, } D_r = \frac{2K}{K-1}M_x^2 - 1 + 1 + \left(\frac{K-1}{2}\right)M_x^2$$

$$= M_x^2\left[\frac{2K}{K-1}+\frac{K-1}{2}\right] = M_x^2\left[\frac{4K+(K-1)^2}{2(K-1)}\right] = M_x^2\,\frac{(K+1)^2}{2(K-1)}$$

Hence $\dfrac{T_y}{T_x} = \dfrac{\left(1+\dfrac{K-1}{2}M_x^2\right)\left(\dfrac{2K}{K-1}M_x^2-1\right)}{\dfrac{(K+1)^2}{2(K-1)}M_x^2}$

(3) Prove that $\dfrac{P_{oy}}{P_{ox}} = \left[\dfrac{\dfrac{K+1}{2}M_x^2}{1+\dfrac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}\left[\dfrac{2}{K+1}KM_x^2-\dfrac{K-1}{K+1}\right]^{\frac{1}{K-1}}$

For the isentropic x – ox,

$$\frac{P_{ox}}{P_x} = \left[1+\frac{K-1}{2}M_x^2\right]^{\frac{K}{K-1}}$$

$$P_{ox} = P_x\left[1+\frac{K-1}{2}M_x^2\right]^{\frac{K}{K-1}}$$

$$P_{oy} = P_y\left[1+\frac{K-1}{2}M_y^2\right]^{\frac{K}{K-1}}$$

$$\frac{P_{oy}}{P_{ox}} = \frac{P_y}{P_x}\left[\frac{1+\dfrac{K-1}{2}M_y^2}{1+\dfrac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}} \quad \text{......... (1)}$$

But $\dfrac{P_y}{P_x}$ is obtained from momentum as

$$\frac{P_y}{P_x} = \frac{1+KM_x^2}{1+KM_y^2}$$

But $M_y^2 = \left[\dfrac{\dfrac{2}{K-1}+M_x^2}{\dfrac{2K}{K-1}M_x^2-1}\right]$

$$\frac{P_y}{P_x} = \frac{1+KM_x^2}{1+K\left[\dfrac{\dfrac{2}{K-1}+M_x^2}{\dfrac{2K}{K-1}M_x^2-1}\right]}$$

$$\frac{P_y}{P_x} = \frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}$$

$$\frac{1+\dfrac{K-1}{2}M_y^2}{1+\dfrac{K-1}{2}M_x^2} = \frac{1+\dfrac{K-1}{2}\left[\dfrac{\dfrac{2}{K-1}+M_x^2}{\dfrac{2K}{K-1}M_x^2-1}\right]}{1+\dfrac{K-1}{2}M_x^2} = \frac{\dfrac{(K+1)^2}{2(K-1)}M_x^2}{\left(1+\dfrac{K-1}{2}M_x^2\right)\left(\dfrac{2K}{K-1}M_x^2-1\right)}$$

$\therefore$ Equation (1) becomes

$$\frac{P_{oy}}{P_{ox}} = \left[\frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}\right]\left[\frac{\dfrac{(K+1)^2}{2(K-1)}M_x^2}{\left(1+\dfrac{K-1}{2}M_x^2\right)\left(\dfrac{2K}{K-1}M_x^2-1\right)}\right]^{\frac{K}{K-1}}$$

$$= \left[\frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}\right]\left[\frac{\dfrac{K+1}{2}M_x^2}{\left(1+\dfrac{K-1}{2}M_x^2\right)\left(\dfrac{2K}{K+1}M_x^2-\dfrac{K-1}{K+1}\right)}\right]^{\frac{K}{K-1}}$$

$$= \left(\frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}\right)^{1-\frac{K}{K-1}}\left[\frac{\dfrac{K+1}{2}M_x^2}{1+\dfrac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}$$

$$\therefore \frac{P_{oy}}{P_{ox}} = \left(\frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}\right)^{\frac{-1}{K-1}}\left[\frac{\dfrac{K+1}{2}M_x^2}{1+\dfrac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}$$

(4) Prove that $\dfrac{V_y}{V_x} = \dfrac{1 - \dfrac{K-1}{2} M_x^2}{\dfrac{K+1}{2} M_x^2}$

$$M_x^2 = \frac{V_x^2}{C_x^2}$$

$$V_x^2 = M_x^2 C_x^2$$

$$V_y^2 = M_y^2 C_y^2$$

$$\frac{V_y^2}{V_x^2} = \frac{M_y^2}{M_x^2} \cdot \frac{T_y}{T_x} \qquad \text{......... (1)}$$

But $\dfrac{T_y}{T_x} = \dfrac{1 + \dfrac{K-1}{2} M_x^2}{1 + \dfrac{K-1}{2} M_y^2}$

But $M_y^2 = \dfrac{\dfrac{2}{K-1} + M_x^2}{\dfrac{2K}{K-1} M_x^2 - 1}$

Substituting and simplifying

$$\frac{T_y}{T_x} = \frac{\left(1 + \dfrac{K-1}{2} M_x^2\right)\left(\dfrac{2K}{K-1} M_x^2 - 1\right)}{\dfrac{(K+1)^2}{2(K-1)} M_x^2} \qquad \text{............ (2)}$$

Substituting (2) in (1), we have

$$\frac{V_y^2}{V_x^2} = \frac{\left[\dfrac{\dfrac{2}{K-1} + M_x^2}{\dfrac{2K}{K-1} M_x^2 - 1}\right]}{M_x^2} \times \frac{\left(1 + \dfrac{K^2-1}{2} M_x^2\right)\left(\dfrac{2K}{K-1} M_x^2 - 1\right)}{\dfrac{(K+1)^2}{2(K-1)} M_x^2}$$

$$= \frac{\left(\frac{2}{K-1} + M_x^2\right)\left(1 + \frac{K-1}{2} M_x^2\right)}{\frac{(K+1)^2}{2(K-1)} M_x^4}$$

$$= \frac{\left(\frac{2}{K-1}\right)\left(1 + \frac{K-1}{2} M_x^2\right)^2}{\frac{(K+1)^2}{2(K-1)} M_x^4} = \frac{\left(1 + \frac{K-1}{2} M_x^2\right)^2}{\frac{(K+1)^2 M_x^4}{4}}$$

$$\therefore \frac{V_y}{V_x} = \frac{1 + \frac{K-1}{2} M_x^2}{\frac{K+1}{2} M_x^2}$$

(5) Prove that $\frac{P_{oy}}{P_x} = \left(\frac{K+1}{2} M_x^2\right)^{\frac{K}{K-1}} \left[\frac{2K}{K+1} M_x^2 - \frac{K-1}{K+1}\right]^{\frac{1}{K-1}}$

(This is known as Rayleigh supersonic Putot tube formula)

We have, $\frac{P_{oy}}{P_y} - \left(1 + \frac{K-1}{2} M_y^2\right)^{\frac{K}{K-1}}$

$$\frac{P_y}{P_x} = \left[\frac{2K}{K+1} M_x^2 - \frac{K-1}{K+1}\right]$$

$$\frac{P_{oy}}{P_x} = \left(1 + \frac{K-1}{2} M_y^2\right)^{\frac{K}{K-1}} \times \left(\frac{2K}{K-1} M_x^2 - \frac{K-1}{K+1}\right)$$

$$= \left\{1 + \frac{K-1}{2}\left[\frac{\frac{2}{K-1} + M_x^2}{\frac{2K}{K-1} M_x^2 - 1}\right]\right\}^{\frac{K}{K-1}} \left(\frac{2K}{K+1} M_x^2 - \frac{K-1}{K+1}\right)$$

$$= \left[\frac{\frac{2K}{K-1} M_x^2 - 1 + 1 + \frac{K-1}{2} M_x^2}{\frac{2K}{K-1} M_x^2 - 1}\right]^{\frac{K}{K-1}} \left(\frac{2K}{K+1} M_x^2 - \frac{K-1}{K+1}\right)$$

$$=\left[\frac{M_x^2\left[\frac{2K}{K-1}+\frac{K-1}{2}\right]}{\frac{2K}{K-1}M_x^2-1}\right]^{\frac{K}{K-1}}\left(\frac{2K}{K+1}M_x^2-\frac{K-1}{K+1}\right)$$

$$=\left[\frac{\frac{(K+1)^2 M_x^2}{2(K-1)}}{\frac{2K}{K-1}M_x^2-1}\right]^{\frac{K}{K-1}}\left(\frac{2K}{K+1}M_x^2-\frac{K-1}{K+1}\right)$$

$$=\frac{\left[\frac{(K+1)M_x^2}{2}\right]^{\frac{K}{K-1}}}{\left[\frac{2K}{K+1}M_x^2-\frac{K-1}{K+1}\right]^{\frac{K}{K-1}}}\times\left(\frac{2K}{K+1}M_x^2-\frac{K-1}{K+1}\right)$$

$$\frac{P_{oy}}{P_x}=\left[\frac{(K+1)M_x^2}{2}\right]^{\frac{K}{K-1}}\times\left[\frac{2K}{K+1}M_x^2-\frac{K-1}{K+1}\right]^{\frac{-1}{K-1}}$$

(6) Show that shocks are possible only in supersonic flow:

$$\Delta S = C_p \ln\frac{T_2}{T_1} - R\ \ln\ \frac{P_2}{P_1}$$

Fro a shock

$$\Delta S = C_p \ln\frac{T_y}{T_x} - R \ln\ \frac{P_y}{P_x}$$

But $\Delta S = \Delta S_o$

$$= C_p\ \ln\ \frac{T_{oy}}{T_{ox}} - R\ \ln\ \frac{P_{oy}}{P_{ox}} = -R\ \ln\ \frac{P_{oy}}{P_{ox}}$$

$$\frac{\Delta S}{R} = \ \ln\left(\frac{P_{oy}}{P_{ox}}\right)^{-1}$$

$$= -\ln\left[\frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}\right]^{\frac{-1}{K-1}}\left[\frac{\frac{K+1}{2}M_x^2}{1+\frac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}$$

A plot of this equation is shown in the Figure.

Fro $M_x < 1$ the entropy change is negative as shown. Since e shocks are highly irreversible in nature, second law insists that the entropy should be produced across a shock. Therefore shocks are possible only in supersonic flow.

Alternatively, $\frac{\Delta S}{C_V}$ can be obtained by expanding the logarithmic tersm interms of $\left(M_x^2 - 1\right)$ and considering terms up to third powers of $\left(M_x^2 - 1\right)$, we get,

$$\frac{\Delta S}{C_v} = \frac{2K(K-1)}{3(K+1)^2}\left(M_x^2 - 1\right)^3$$

From the above equation for $M_x > 1$, $\frac{\Delta S}{C_V}$ becomes positive. Therefore shocks are possible only in supersonic flows.

Prandtl-Meyer relationship

$V_x . V_y = C^{*2}$ or

$M_x^2 M_y^2 = 1$

Governing relations for a normal shock

(1) Continuity

$$\rho_x V_x = \rho_y V_y \qquad \text{....... (1)}$$

(2) Energy

$$h_{ox} = h_{oy}$$

$$h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2} = h_0 \qquad \text{............ (2)}$$

(3) Momentum

$$\Sigma F_{xx} = \frac{\partial}{\partial t}\left(\dot{m}V_{xx}\right)_{CV} + \left(\dot{m}V_{xx}\right)_{out} - \left(\dot{m}V_{xx}\right)_{in}$$

$$P_x A - P_y A = \dot{m}\left(V_y - V_x\right)$$

$$\left(P_x - P_y\right) = G\left(V_y - V_x\right)$$

$$P_x - P_y = \rho V\left(V_y - V_x\right) \qquad \text{............. (3)}$$

From (2) we have

$$h_x + \frac{V_x^2}{2} = h_0$$

$$C_p T_x + \frac{V_x^2}{2} = C_p T_o$$

$$\frac{KR}{K-1}T_x + \frac{V_x^2}{2} = \frac{KR}{K-1}T_0$$

$$\frac{C_x^2}{K-1} + \frac{V_x^2}{2} = \frac{C_0^2}{K-1}$$

$$\therefore \; C_x^2 = C_0^2 - \frac{K-1}{2}V_x^2 \qquad \text{.......... (4)}$$

Similarly $C_y^2 = C_0^2 - \frac{K-1}{2}V_y^2$ .... (5)

Equation (3) $\Rightarrow P_x - P_y = \rho V\left(V_y - V_x\right)$

$$\frac{P_x - P_y}{\rho V} = \left(V_y - V_x\right)$$

$$\frac{P_x}{\rho_x V_x} - \frac{P_y}{\rho_y V_y} = \left(V_y - V_x\right)$$

$$\frac{RT_x}{V_x} - \frac{RT_y}{V_y} = \left(V_y - V_x\right)$$

$$\frac{KRT_x}{V_x} - \frac{KRT_y}{V_y} = K\left(V_y - V_x\right)$$

$$\frac{C_x^2}{V_x} - \frac{C_y^2}{V_y} = K\left(V_y - V_x\right) \qquad \text{............. (6)}$$

Using (4) and (5) in (6)

$$\frac{1}{V_x}\left[C_0^2 - \frac{K-1}{2}V_x^2\right] - \frac{1}{V_y}\left[C_0^2 - \frac{K-1}{2}V_y^2\right] = K\left(V_y - V_x\right)$$

$$C_0^2\left[\frac{1}{V_x} - \frac{1}{V_y}\right] + \frac{K-1}{2}\left(V_y - V_x\right) = K\left(V_y - V_x\right)$$

$$\frac{C_0^2\left(V_y - V_x\right)}{V_x V_y} + \frac{K-1}{2}\left(V_y - V_x\right) = K\left(V_y - V_x\right)$$

$$\frac{C_0^2}{V_x V_y} = K - \frac{K-1}{2}$$

$$C_0^2 = \left(\frac{K+1}{2}\right)V_x V_y \qquad \text{.......... (7)}$$

$$\frac{C^{*2}}{C_0^2} = \frac{KRT^*}{KRT_0} = \frac{T^*}{T_0} = \frac{2}{K+1} \qquad \left[\frac{T_0}{T} = 1 + \frac{K-1}{2}M^2\right]$$

$$\left[\frac{T_0}{T^*} = \frac{K+1}{2}\right]$$

$$C_0^2 = \left(\frac{K+1}{2}\right)C^{*2} \qquad \text{............ (8)}$$

Substitution (8) in (7) we have

$$\left(\frac{K+1}{2}\right)C^{*2} = \left(\frac{K+1}{2}\right)V_x V_y$$

$$C^{*2} = V_x V_y$$

$$\frac{V_x V_y}{C^{*2}} = 1$$

for a shock, $C_x^* = C_y^* = C^*$

$$\frac{V_x}{C_x^*}\frac{V_y}{C_y{}^*} = 1$$

$M_x^* . M_y^* = 1$ By definition $M^* = \dfrac{V}{C^*}$

THE RANKINE – HUGONIOT EQUATIOS

DENSITY RATIO ACROSS THE SHOCK

We know that density, $\rho = \dfrac{P}{RT}$

For upstream shock

$$\rho x = \frac{P_x}{R_x T_x}$$

For down stream shock

$$\rho_y = \frac{P_y}{R_y T_y}$$

$$\frac{\rho_y}{\rho x} = \frac{\dfrac{P_y}{R_y T_y}}{\dfrac{P_x}{R_x T_x}}$$

$$\frac{\rho_y}{\rho x} = \frac{p_y}{p_x} \times \frac{T_X}{T_Y}$$

We know that,

$$\boxed{\frac{P_Y}{P_X}=\frac{2\gamma}{\gamma+1}M_X{}^2-\left(\frac{\gamma-1}{\gamma+1}\right)}$$

$$\frac{2\gamma}{\gamma+1}M_X{}^2=\frac{P_Y}{P_X}+\frac{\gamma-1}{2\gamma}$$

we know that

$$\frac{T_Y}{T_X}=\frac{\left[\frac{2\gamma}{\gamma+1}M_X{}^2-1\right]\left[1+\frac{\gamma-1}{2}M_X{}^2\right]}{\frac{M_X{}^2}{2(\gamma-1)}\times(\gamma+1)^2}$$

$$\frac{T_Y}{T_X}=\frac{\frac{2\gamma}{\gamma-1}\left[\frac{\gamma+1}{2\gamma}\left(\frac{P_Y}{P_X}\right)+\frac{\gamma-1}{2\gamma}\right]-1\left[1+\frac{\gamma-1}{2}\left(\frac{\gamma+1}{2\gamma}\left(\frac{P_Y}{P_X}\right)+\frac{\gamma-1}{2\gamma}\right)\right]}{\left[\frac{\gamma+1}{2\gamma}\left(\frac{P_Y}{P_X}\right)+\frac{\gamma-1}{2\gamma}\right]\times\frac{(\gamma+1)^2}{2(\gamma-1)}}$$

$$\frac{T_Y}{T_X}=\frac{\left[\frac{(\gamma+1)}{(\gamma-1)}\frac{P_Y}{P_X}+1-1\right]\times\left[1+\frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{P_Y}{P_X}+\frac{(\gamma-1)^2}{4\gamma}\right]}{\frac{(\gamma-1)^3}{4\gamma(\gamma-1)}\left(\frac{P_Y}{P_X}\right)+\frac{(\gamma-1)^2}{4\gamma}}$$

Taking out $\frac{(\gamma-1)^3}{4\gamma(\gamma-1)}\left(\frac{P_Y}{P_X}\right)$

$$=\frac{\frac{(\gamma-1)^3}{4\gamma(\gamma-1)}\left(\frac{P_Y}{P_X}\right)\left[\frac{4\gamma}{(\gamma+1)^2}+\frac{(\gamma-1)}{(\gamma+1)}\frac{P_Y}{P_X}+\frac{(\gamma-1)^2}{(\gamma+1)^2}\right]}{\frac{(\gamma-1)^3}{4\gamma(\gamma-1)}\left(\frac{P_Y}{P_X}+\frac{(\gamma-1)}{(\gamma+1)}\right)}$$

$$=\frac{\left(\frac{P_Y}{P_X}\right)\left[\frac{4\gamma}{(\gamma+1)^2}+\frac{(\gamma-1)}{(\gamma+1)}\frac{P_Y}{P_X}+\frac{(\gamma-1)^2}{(\gamma+1)^2}\right]}{\left(\frac{P_Y}{P_X}+\frac{(\gamma-1)}{(\gamma+1)}\right)}$$

$$=\frac{\frac{P_Y}{P_X}\left[\frac{(\gamma-1)}{(\gamma+1)}\times\frac{P_Y}{P_X}+\frac{4\gamma+(\gamma-1)^2}{(\gamma+1)^2}\right]}{\frac{(\gamma-1)}{(\gamma+1)}+\frac{P_Y}{P_X}}$$

$$=\frac{\frac{P_Y}{P_X}\left[\left(\frac{\gamma-1}{\gamma+1}\right)\times\frac{P_Y}{P_X}+\frac{4\gamma+(\gamma^2-2\gamma+1)^2}{(\gamma+1)^2}\right]}{\frac{P_Y}{P_X}+\frac{\gamma-1}{\gamma+1}}$$

$$=\frac{\frac{P_Y}{P_X}\left[\frac{(\gamma-1)}{(\gamma+1)}\times\frac{P_Y}{P_X}+\frac{(\gamma^2+2\gamma+1)}{(\gamma+1)^2}\right]}{\frac{P_Y}{P_X}+\frac{\gamma-1}{\gamma+1}}$$

$$=\frac{\frac{P_Y}{P_X}\left[\frac{(\gamma-1)}{(\gamma+1)}\times\frac{P_Y}{P_X}+\frac{(\gamma+1)^2}{(\gamma+1)^2}\right]}{\frac{P_Y}{P_X}+\frac{\gamma-1}{\gamma+1}}$$

$$\frac{T_Y}{T_x}=\frac{\frac{P_Y}{P_X}\left[\frac{(\gamma-1)}{(\gamma+1)}\times\frac{P_Y}{P_X}+1\right]}{\frac{P_Y}{P_X}+\frac{\gamma-1}{\gamma+1}}$$

$$\frac{P_Y}{P_X}=\frac{\frac{T_Y}{T_x}\times\left(\frac{P_Y}{P_X}+\frac{\gamma-1}{\gamma+1}\right)}{\left[1+\frac{P_Y}{P_X}\times\frac{\gamma-1}{\gamma+1}\right]}$$

$$\frac{P_Y}{P_X}\times\frac{T_x}{T_Y}=\frac{\dfrac{P_Y}{P_X}+\dfrac{\gamma-1}{\gamma+1}}{1+\dfrac{\gamma-1}{\gamma+1}\times\dfrac{P_Y}{P_X}}$$

We have

$$\frac{\rho_y}{\rho_x}=\frac{P_Y}{P_X}\times\frac{T_x}{T_Y}$$

$$\frac{\rho_y}{\rho_x}=\frac{\dfrac{P_Y}{P_X}+\dfrac{\gamma-1}{\gamma+1}}{1+\dfrac{\gamma-1}{\gamma+1}\times\dfrac{P_Y}{P_X}}$$

$$\frac{\rho_y}{\rho_x}=\frac{\dfrac{\gamma-1}{\gamma+1}\left[1+\dfrac{\gamma-1}{\gamma+1}\times\dfrac{P_Y}{P_X}\right]}{\dfrac{\gamma-1}{\gamma+1}\left[\dfrac{\gamma-1}{\gamma+1}+\dfrac{P_Y}{P_X}\right]}$$

$$\boxed{\frac{\rho_y}{\rho_x}=\frac{\left[1+\dfrac{\gamma+1}{\gamma-1}\times\dfrac{P_Y}{P_X}\right]}{\left[\dfrac{\gamma+1}{\gamma-1}+\dfrac{P_Y}{P_X}\right]}}$$

$$\frac{\rho_y}{\rho_x}\left[\frac{\gamma+1}{\gamma-1}\times\frac{P_Y}{P_X}\right]=\left[1+\frac{\gamma+1}{\gamma-1}\times\frac{P_Y}{P_X}\right]$$

$$\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)+\left(\frac{\rho_y}{\rho_x}\right)\frac{P_Y}{P_X}=1+\frac{\gamma+1}{\gamma-1}\times\frac{P_Y}{P_X}$$

$$\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)=1+\frac{\gamma+1}{\gamma-1}\times\frac{P_Y}{P_X}-\left(\frac{\rho_y}{\rho_x}\right)\frac{P_Y}{P_X}$$

$$\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)=1+\frac{P_Y}{P_X}\left[\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}\right]$$

$$\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)-1=\frac{P_Y}{P_X}\left[\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}\right]$$

$$\boxed{\frac{P_Y}{P_X}=\frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)-1}{\left[\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}\right]}}$$

the above eqn.s is knows Rankine – Hugoniot equations

Strength of a Shock Wave

It is defined as the ratio of difference in down stream and upstream shock pressures (p_y - p_x) to upstream shock pressures (p_x). It is denoted by ξ.

$$\xi=\frac{p_y-p_x}{p_x}$$

$$\boxed{\xi=\frac{p_y}{p_x}-1}$$

Substituting for p_y/p_x

$$\xi = \left[\frac{2\gamma}{\gamma+1} M_x{}^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right] - 1$$

$$= \frac{1}{\gamma+1}\left[2\gamma M_x{}^2 - (\gamma-1) - (\gamma+1)\right]$$

$$= \frac{1}{\gamma+1}\left[2\gamma M_x{}^2 - \gamma + 1 - \gamma - 1\right]$$

$$= \frac{1}{\gamma+1}\left[2\gamma M_x{}^2 - 2\gamma\right]$$

$$\boxed{\xi = \frac{2\gamma}{\gamma+1}[M_x{}^2 - 1]}$$

From the above equation;

$$\boxed{\xi \propto M_x{}^2 - 1}$$

The strength of shock wave may be expressed in another form using Rankine-Hugoniot equation.

$$\frac{p_y}{p_x} = \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)-1}{\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}}$$

We know that

$$\xi = \frac{p_y}{p_x} - 1$$

$$\Rightarrow \quad \xi = \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)-1}{\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}} - 1$$

$$= \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)-1-\frac{\gamma+1}{\gamma-1}+\frac{\rho_y}{\rho_x}}{\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}}$$

$$= \frac{\frac{\gamma+1}{\gamma-1}\left[\frac{\rho_y}{\rho_x}-1\right]+\left[\frac{\rho_y}{\rho_x}-1\right]}{\frac{\gamma+1+1-1}{\gamma-1}-\frac{\rho_y}{\rho_x}}$$

$$= \frac{\frac{\gamma+1}{\gamma-1}\left[\frac{\rho_y}{\rho_x}-1\right]+\left[\frac{\rho_y}{\rho_x}-1\right]}{\frac{\gamma-1}{\gamma-1}+\frac{2}{\gamma-1}-\frac{\rho_y}{\rho_x}}$$

$$= \frac{\left[\frac{\rho_y}{\rho_x}-1\right]\left[\frac{\gamma+1}{\gamma-1}+1\right]}{\frac{2}{\gamma-1}+1-\frac{\rho_y}{\rho_x}}$$

$$= \frac{\left[\frac{\rho_y}{\rho_x}-1\right]\left[\frac{\gamma+1+\gamma-1}{\gamma-1}\right]}{\frac{2}{\gamma-1}-\left[\frac{\rho_y}{\rho_x}-1\right]}$$

$$= \frac{\left[\frac{\rho_y}{\rho_x}-1\right]\left[\frac{2\gamma}{\gamma-1}\right]}{\frac{2}{\gamma-1}-\left[\frac{\rho_y}{\rho_x}-1\right]}$$

$$\Rightarrow \boxed{\xi = \frac{\left[\frac{2\gamma}{\gamma-1}\right]\left[\frac{\rho_y}{\rho_x}-1\right]}{\frac{2}{\gamma-1}-\left[\frac{\rho_y}{\rho_x}-1\right]}}$$

From this equation we came to know strength of shock wave is directly proportional to;

$$\left[\frac{\rho_y}{\rho_x}-1\right]$$

$$\Rightarrow \boxed{\xi \propto \left[\frac{\rho_y}{\rho_x}-1\right]}$$

PROBLEMS

Que.1

The state of a gas (γ **=1.3,R =0.469 KJ/KgK.) upstream of normal shock wave is given by the following date:**

M_x =2.5 , P_x =2 bar. T_x =275 K calculate the Mach number ,pressure,temperatureand velocity of a gas down stream of shock: check the calculated values with those given in the gas tables.

Take K = γ

$$M_y^2 = \frac{\dfrac{2}{K-1}+M_x^2}{\dfrac{2K}{K-1}M_x^2-1} = \frac{\dfrac{2}{1.3-1}+2.5^2}{\dfrac{2\times 1.3}{1.3-1}\times 2.5^2-1} = \frac{12.92}{53.19} = 0.243$$

$$M_Y = 0.4928$$

$$\boxed{\frac{P_Y}{P_X} = \frac{2\gamma}{\gamma+1}M_X{}^2 - \left(\frac{\gamma-1}{\gamma+1}\right)}$$

$$= \frac{2\times 1.3}{1.3+1}\times 1.5^2 - \left(\frac{1.3-1}{1.3+1}\right)$$

$$\frac{P_Y}{P_X} = 7.065 - 0.130 = 6.935$$

$$P_Y = 6.935 \times 2 = 13.870 \ bar$$

$$\frac{T_y}{T_x} = \frac{\left(1 + \frac{K-1}{2} M_x^2\right)\left(\frac{2K}{K-1} M_x^2 - 1\right)}{\frac{(K+1)^2}{2(K-1)} M_x^2}$$

$$= \frac{\left(1 + \frac{1.3-1}{2} \times 2.5^2\right)\left(\frac{2 \times 1.3}{1.3-1} 2.5^2 - 1\right)}{\frac{(1.3+1)^2}{2(1.3-1)} 2.5^2}$$

$$= \frac{\left(1 + \frac{03}{2} \times 6.25\right) \times 53.19}{\frac{(2.3)^2}{2(0.3)} \times 6.25} = \frac{1.937 \times 53.19}{55.104}$$

$$\frac{T_y}{T_x} = 1.869$$

$$T_Y = 1.869 \times 275 = 513.975 \mathrm{K}$$

$$\frac{C_Y}{C_x} = \frac{3}{1.3+1} \times \frac{1}{6.25} + \frac{0.3}{2.3} = 0.269$$

$$C_Y = 0.269 C_x \quad .0269 M_x a_x$$

$$C_Y = 0.269 M_x \sqrt{KRT_x}$$

$$C_Y = 0.269 \, 2.5_x \sqrt{1.3 \times 469 \times 275}$$

$$Alternately, \quad C_y = M_y \sqrt{KRT_y}$$

$$= 0.4928 \sqrt{1.3 \times 469 \times 513.975}$$

$$C_y = 275.16 \, m/s.$$

A gas(γ **=1.3,R =0.287KJ/KgK) at a Mach number of 1.8 P =0.8 bar and T=373K pass through a normal shock .Compare this value in an isentropic compression through the same pressure ratio.**

$$\rho = \frac{P_x}{RT_x} = \frac{0.8\times10^5}{287\times373} = 0.747\,Kg/m^3$$

$$From\ normal\ shock\ table\ for\ \gamma = 1.4\ at\ M_x = 1.8$$

$$\frac{P_Y}{P_x} = 3.613\,,\ \frac{T_y}{T_x} = 1.532$$

$$\frac{\rho_y}{\rho x} = \frac{p_y}{p_x}\times\frac{T_X}{T_Y}$$

$$\frac{\rho_y}{\rho x} = \frac{3.613}{1.532} = 2.358$$

$$\rho_y = 2.358\times0.747 = 1.762\,Kg/m^3$$

$$For\ isentropic\ flow$$

$$\frac{\rho_y}{\rho x} = \left(\frac{p_y}{p_x}\right)^{\frac{1}{\gamma}}$$

$$\frac{\rho_y}{\rho x} = (3.613)^{\frac{1}{1.4}} = 2.5$$

$$\rho_y = 2.5\times0.747 = 1.867\,Kg/m^3$$

It is noted that the final density of the isentropic process is grater than in the shock process.

Q.A jet of air at 270 K and 0.2 bar has an intial Mach number of 1.9.If the process through a normal shock wave. Determine the following for the down stream of the shock.

1.Mach number.= M_y

2. Pressure. = P_y

3. Temperature = T_y.

4.Speed of sound .= a_y

5. Jet velocity = C_y

6. Density .= ρ_y

Given.

T_x =270 K

P_x =0.7 bar =0.7×10^5 N/m^2

From normal shock table for M_x=1.9 and γ =1.4

$$M_Y = 0.596$$

$$\frac{P_Y}{P_x} = 4.045$$

$$\frac{T_y}{T_x} = 1.608$$

$$P_y = 4.045\times P_x$$

$$= 4.045\times 0.7\times 10^5$$

$$P_y = 2.831\times 10^5$$

$$T_y = 1.608\times T_x$$

$$= 1.608\times 270$$

$$= 434.16\,\mathrm{K}$$

Speed of at down of the shock

$$a_y = \sqrt{\gamma R T_Y}$$

$$= \sqrt{1.4\times 287\times 434.16}$$

$$a_y = 417.66\, m/s$$

Jet velocity at downstream of the shock

$$C_y = M_Y \times a_y$$

$$=0.596 \times 417.66$$

$$C_y = 248.93\, m/s$$

$$Density = \rho_y = \frac{P_y}{RT_y} = \frac{2.83 \times 10^5}{287 \times 434.16} = 2.27\, kg/m^3$$

Q.An Aircraft flies at a Mach number of 1.1 at an altitude of 15,000 metres.The compression in its engine is partially achieved by a normal shock wave standing at the entry of the diffuser. Determine the following for downstream of the shock.

1. Mach number

2. Temperature of the air

3. Pressure of the air

4. Stagnation pressure loss across the shock.

Given

$$M_x = 1.1$$

$$Altitude, Z = 15,000\, m$$

To find

1. M_y

2. T_y

3. P_y

4. $\Delta P_0 = P_{Ox} - P_{Oy}$

Refer gas tables for Altitude, $Z = 15,000\, m$

$$T_x = 216.6\, K$$

$$P_x = 0.120\, bar$$

$$P_x = 0.120 \times 10^5\, N/m^2$$

$\text{Re } fer\ Normal\ shocks\ gas\, tables\ for\ M_x = 1.1\ and\, \gamma = 1.4$

$$M_y = 0.911$$

$$\frac{P_y}{P_x} = 1.245$$

$$\frac{T_y}{T_x} = 1.065$$

$$\frac{P_{0y}}{P_{0x}} = 0.998$$

$$\frac{P_{0y}}{P_x} = 2.133$$

$$P_y = 1.24 \times 5 P_x$$
$$= 1.24 \times 0.120 \times 10^5\, N/m^2$$

$$P_y = 0.149 \times 10^5\, N/m^2$$

$$T_y = 1.067 \times T_x$$
$$= 1.065 \times 216.6$$

$$T_y = 230.67 K$$

$$P_{0y} = 2.133 \times P_x$$
$$= 2.133 \times 0.120 \times 10^5$$

$$P_{0y} = 0.259 \times 10^5\, N/m^2$$

$$P_{0x} = \frac{P_{0y}}{0.998}$$

$$P_{0x} = \frac{0.259 \times 10^5}{0.998}$$

$P_{0x} = 0.2564 \times 10^5 \, N/m^2$

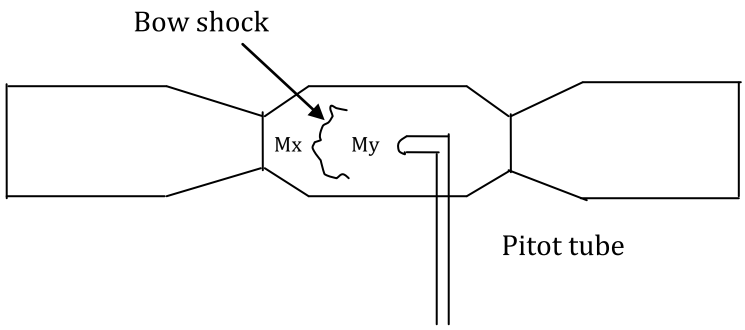
Pr*essure loss*

$\Delta P_0 = P_{Ox} - P_{Oy}$

$= 0.2564 \times 10^5 - 0.259 \times 10^5$

$\Delta P_0 = 50 N/m^2$

1) **When a pitot tube is immersed in a supersonic stream a normal shock is formed ahead of the Pitot tube mouth. After the shock the fluid stream decelerates isentropically to the total pressure of the entrance to the pitot tube. A pitot tube travelling in a supersonic wind tunnel gives values of 15Kpa and 70Kpa for the static pressure upstream of the shock and the pressure at the inlet of the tube respectfully. Find the Mach no. of the tunnel if the stagnation temperature is 575K. Calculate the static temperature and the total (stagnation) pressure upstream and the downstream of the tube.**



$\mathbf{P_x = 15 \times 10^3 \ pa = 15 \times 10^3 \ N/m^2}$

$\mathbf{Poy = 70 \times 10^3 \ N/m^2}$

$\mathbf{T_0 = T_{0x} = T_{0y} = 575 \ K}$

Refer Normal shock tables for $\frac{P_{0y}}{P_x} = 4.67$, **and γ = 1.4**

$$\frac{P_{0y}}{P_x} = \frac{70 \times 10^3}{15 \times 10^3} = 4.375$$

$M_x = 1.8$
$M_y = 0.1616$

$$\frac{P_y}{P_x} = 3.613$$

$$\frac{T_y}{T_x} = 1.532$$

$$\frac{P_{0y}}{P_{0x}} = 0.813$$

$$P_{0x} = \frac{78 \times 10^3}{0.1813}$$

Pox = 0.86 × 10^5 N/m^2

we know that,

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{T_{0x}}{T_x} = 1 + \frac{\gamma - 1}{2} M_x{}^2$$

$$\frac{575}{T_x} = 1 + \frac{1.4 - 1}{2} \times 1.8^2$$

$T_x = 348.9\,K$

From table,

$$\frac{T_y}{T_x} = 1.532$$

$T_y = 1.532 \times 348.9 = 534.51\,K$

Result:

M_x = 1.8

M_y = 0.616

T_x = 348.9K

T_y = 534.51K

P_{0x} = 0.86 ×10^5 N/m^2

P0y = 70 ×10^3

Supersonic nozzle is provided with a constant diameter circular duct at its exit. The duct diameter is same as the nozzle diameter. Nozzle exit cross ection is three times that of its throat. The entry conditions of the gas (γ=1.4, R= 287J/KgK) are P_0 =10 bar, T_0 =600K. Calculate the static pressure, Mach number and velocity of the gas in duct.

(a) When the nozzle operates at its design condition.

(b) When a normal shock occurs at its exit.

(c) When a normal shock occurs at a section in the diverging part where the area ratio, A/A* =2.

Given:

A_2 =3A*

Or A_2/A*=3

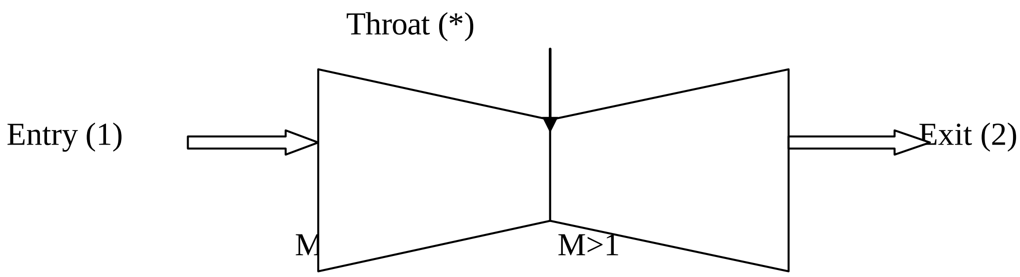
γ = 1.4

R= 287 J/KgK

P_0= 10 bar = 10^6 Pa

T_0= 600K

For nozzle



Solution:

Case (i)

Refer isentropic flow table for A_2/A*=3 and γ= 1.4

M_2 = 2.64

T_2/T_{02}= 0.417

$p_2/ p_{02}= 0.0417$

{Note: For $A_2/A^* =3$, we can refer gas tables page no.30 and 36. But we have to take M > 1 corresponding values since the exit is divergent nozzle}

i.e. $T_2 = 0.417 \times T_{02}$

$= 0.417 \times 600$ {since $T_0=T_{01}=T_{02}$}

$T_2 = 250.2$ K

$p_2 = 0.0471 \times p_{02}$

$= 0.0471 \times 10 \times 10^5$ {since $p_0 =p_{01}= p_{02}$}

$p_2 = 0.471 \times 10^5$ N/m^2

We know

$C_2= M_2\times a_2$

$= M_2 \times \sqrt{\gamma R T_2}$

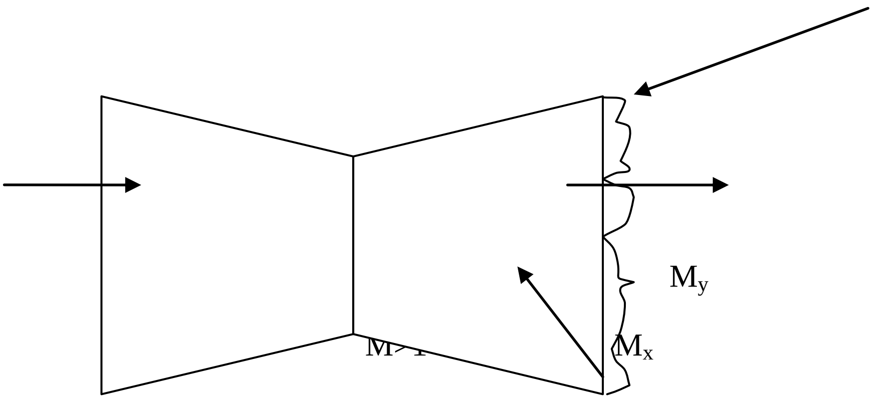
$= 2.64\times (1.4\times 287\times 250.2)^{.5}$

$C_2 =837.05$ m/s

Case (ii)

Normal shock occurs at exit. Shock wave

Entry



$A_2/A^* = A_x/A_x^* =3$ [since in this case $A_2 =A_x$]

Refer isentropic table for $A_x/A_x^* =3$ and $\gamma =1.4$.

$M_x =2.64$

$T_x/T_x^* = 0.417$

$P_x / P_{0x} = 0.0417$ [From gas tables page no. 36]

i.e. $T_x = 0.417 \times T_{0x}$

$= 0.417 \times 600$

i.e. $T_x = 250.2$ K [Since $T_0 = T_{0x}$]

So $p_x = 0.0417 \times P_{0x}$

$= 0.0417 \times 10 \times 10^5$ [Since $p_0 = p_{0x}$]

i.e. $p_x = 0.417 \times 10^5$ N/m^2

Refer Normal shock table for $M_x = 2.64$ and $\gamma = 1.4$

$M_y = 0.5$

$P_y / P_x = 7.965$

$T_y/T_x = 2.279$

i.e. $P_y = 7.695 \times p_x$

$= 7.695 \times 0.471 \times 10^5$

So $p_y = 3.75 \times 10^5$ N/m^2

Also $T_y = 2.279 \times T_x$

$= 2.279 \times 250.2$

i.e. $T_y = 570.2$ K

Also $C_y = M_y \times a_y$

$= M_Y \times \sqrt{\gamma R T_Y}$

$= 0.5 \times (1.4 \times 287 \times 570.2)$

$C_y = 239.32$ m/s

Case (iii)

Area ratio $A/A^* = 2$

i.e. $A_x/A_x^* = 2$

Refer isentropic flow table for $A_x/A_x^* = 2$ and $\gamma = 1.4$

$M_x = 2.2$ [from gas tables page no. 35]

Refer Normal shocks table for $M_x = 2.2$ and $\gamma = 1.4$

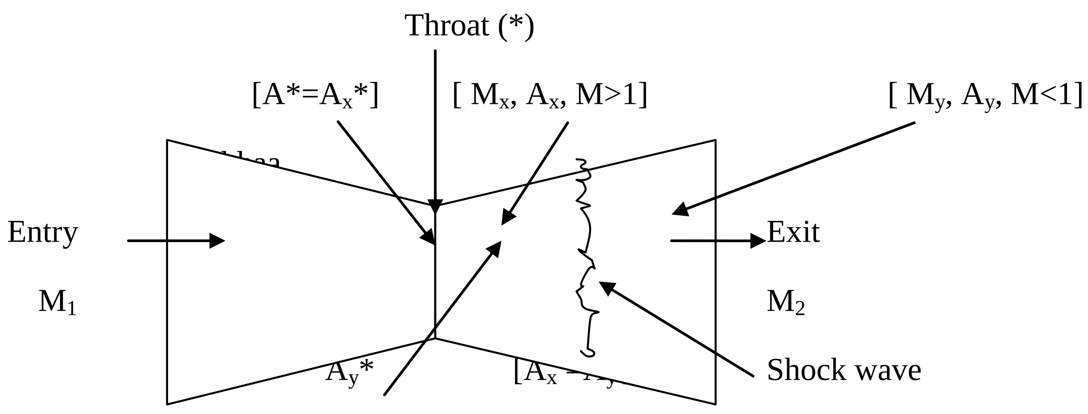
$M_y = 0.547$

$P_{0y}/P_{0x} = 0.628$

$P_{0y} = 0.628 \times P_{0x}$

$= 0.628 \times 10 \times 10^5$

$P_{0y} = 6.28 \times 10^5$ N/m^2



Convergent – Divergent nozzle

Refer isentropic flow table for $M_y = 0.547$ and $\gamma = 1.4$

$A_y/A_y^* = 1.255$

We know that [Since $A_x = A_y$, $A^* = A_x^*$]

$A_2/A_y^* = (A_2/A_x^*) \times (A_x^*/A_x) \times (A_y/A_y^*)$

i.e. $A_2/A_y^* = 3 \times .5 \times 1.255$

or $A_2/A_y^* = 1.8825$

Refer isentropic table for $A_2/A_y^* = 1.8825 = 1.871$ and $\gamma = 1.4$

$M_2 = 0.33$ [From gas tables page no. 29]

$T_2/T_{0y} = 0.978$

$P_2/p_{0y} = 0.927$

i.e. $T_2 = 0.978 \times T_{0y}$

$= 0.978 \times 600$

or $T_2 = 586.8$ K

So $P_2 = 0.927 \times p_{0y}$

$= 0.927 \times 6.28 \times 10^5$

or $P_2 = 5.82 \times 10^5$ N/m^2

$C_2 = M_2 \times a_2$

$= M_2 \times \sqrt{\gamma R T_2}$

$= 0.33 \times (1.4 \times 287 \times 586.8)^{0.5}$

$C_2 = 160.23$ m/s

RESULT

Case (i): $p_2 = 0.471 \times 10^5$ N/m^2, $M_2 = 2.64$, $c_2 = 837.05$ m/s

Case (ii): $p_y = 3.75 \times 10^5$ N/m^2, $M_y = 0.5$, $c_y = 239.32$ m/s

Case (iii): $p_2 = 5.82 \times 10^5$ N/m^2, $M_2 = 0.33$, $c_2 = 160.23$ m/s

The following refers to a supersonic wind tunnel Nozzle throat area- 200 cm^2

Test section cross section -337.5 cm^2 Working fluid is air. Determine the test section mach no. and diffuser throat area if normal shock is located at test section?

Given:

Nozzle throat area = $A_x^* = 200cm^2$

$= 200\times10^{-4} \quad m^2$

Test section area $= A_x = 337.5\ cm^2 = 337\times 10^{-4} m^2$

To find:

1. Test section Mach no. M_x

2. Diffuser throat area, A_y^*

Solution:

$$\frac{A_x}{A^*_x} = \frac{337.5\times10^{-4}}{200\times10^{-4}} = 1.6875$$

Refer isentropic table γ=1.4

Then $M = M_x = 2$

Refer normal shock table for M_x= 2 and γ =1.4

$$\frac{P_{0y}}{P_{0x}} = 0.721$$

We know that $A_x * P_{0x} = A_Y * P_{0y}$

$$A_Y * = A_x * \left(\frac{P_{0x}}{P_{0y}}\right)$$

$$= 200\times10^{-4}\left(\frac{1}{0.721}\right)$$

$$A_Y * = 0.0277 m^2$$

Result:

Test secton Mach number = 1.97

Diffuser throat area = .0272 m^2

ROCKET PROPULSION

What is meant by a jet propulsion system?

It is the propulsion of a jet aircraft (or) Rocket engines which do not use atmospheric air other missiles by the reaction of jet coming out with high velocity. The jet propulsion in used when the oxygen is obtained from the surrounding atmosphere.

Composite propellants (Heterogeneous)

This is a heterogeneous mixture of an oxidiser in a matrix of organic residuous fuel binder. The oxidisers used are usually perchlorates or nitrates. Some of the available fuel binders are polysulfide polyester, epoxy resin synthetic rubbers, polyurethane, polyyisobutane polyisobutane and phenolic of cellulose resins. Many composite propellants are less hazardous to manufacture and handle than double base propellants.

Typical compositions.

Vinylpolyester	Polysulfide	Polyarchane
NH4NO_3-72.5%	NH_4CLO_4-71%	$KCIO_4$-12%
Binder – 25.5%	Binder – 26%	NH_4CLO_4-70%
Additives – 2%	Additives – 1%	Additives – 0.5%
Polybutadiene copolymer	Rubber	
NH_4ClO_4 – 68%	NH_4NO_3-86.5%	
Aluminium-16%	Binder -11.3%	
Binder – 16%	Catalyst – 2.2%	

Processing of Propellants

Fuel and oxidiser are powdered and added in the required proportion and made into a slurry and worked upon to get the required grain shape. There are three methods for processing of Extension by costing processing. The grains are then mechanical to get any required shape.

Propellant Grain and Grain Configuration

The grain is the shaped mass of propellant inside the rocket motor. The propellant material and geometrical configuration of the grain determine the motor performance character sties. The propellant grain is a cast molded or extruded body and its appearance and feel is similar to hard rubber or plastic. Once ignited it will burn on all its exposed surfaces to form hot gases that are then exhausted through a nozzle. Some motors can have more than one grain inside a single case or chamber and some grains have segments made of different propellant composition. However, most rockets have a single grain. The grain must maintain structural integrity during shipment storage and rocket operation.

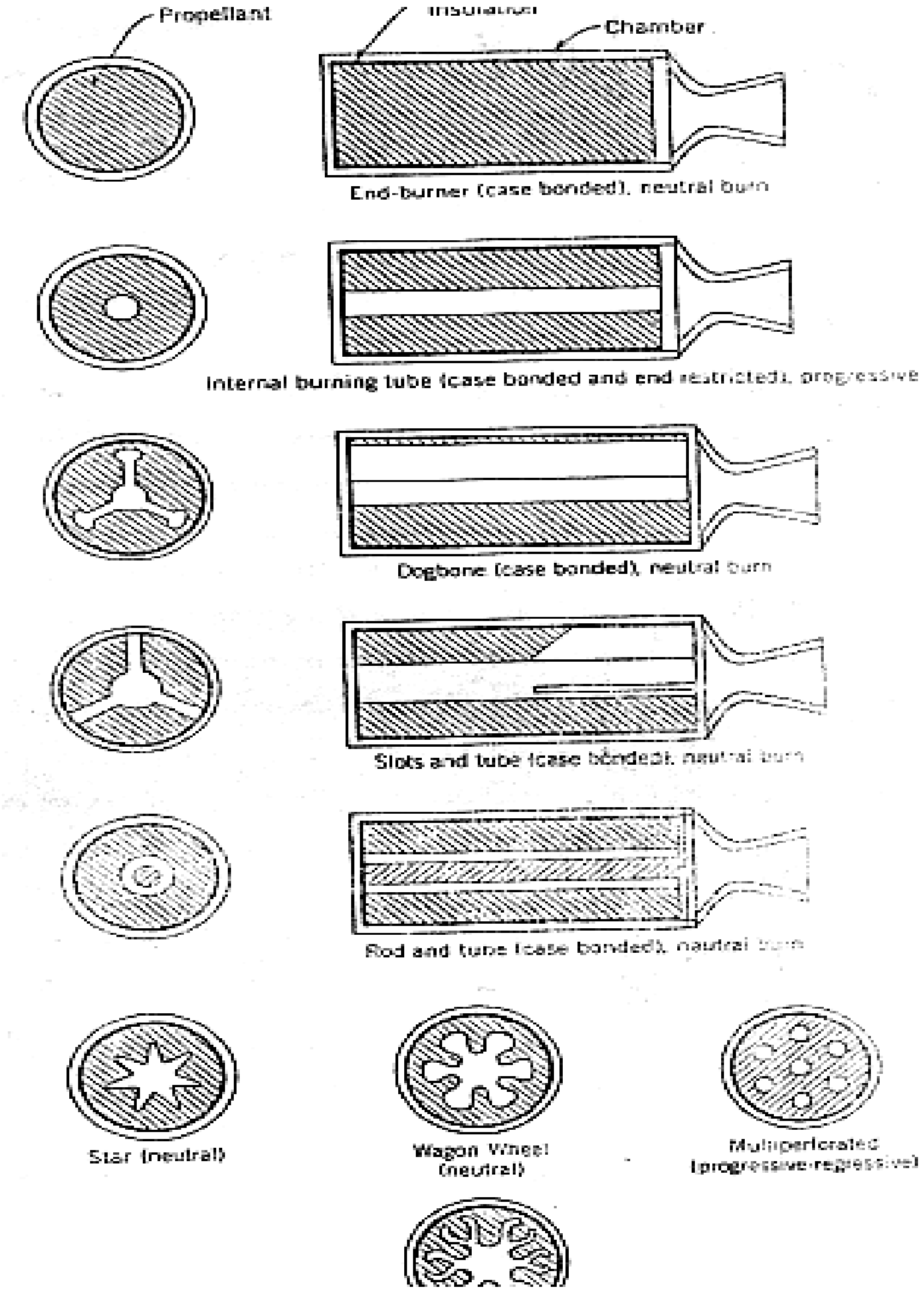
Usually vehicle performance requirements (thrust-time profile) and propellant burning rate and burning surface area are the primary factors governing the selection of a grain geometrical configuration. Processing capability and manufacturing costs can also be influential. More specifically the configuration must satisfy the need of thrust, burning time, chamber pressure regulation total impulse, centre of gravity travel structural strength, shelf-life, temperature limits and manufacturing many grain configurations are available to motor designers (see FIGURE). Grains are generally classified into three types.

According to the thrust action (pressure time characteristics) the grains are classified into

1. Neutral burning grain: Thrust remains essentially constant during the burn period
2. Progressive burning grain: Thrust increases with time.
3. Regressive burning grain: Thrust decreases with time.

Desirable Properties of Solid Propellants

1. A high release of chemical energy- this results in high value of combustion temperature (T_o) and specific impulse or better performance.
2. Low molecular weight (M) of the combustion products – results in higher specific impulse.



$I_{sp} \alpha \ (T_c/M)^{1/2}$

3. Should be stable for a long period of time and should not deteriorate chemically or physically during storage.

4. High density – results in small chamber volume and less chamber weight.

5. Should be unaffected by atmosphere conditions i.e. not hygroscopic.

6. Should be subjected to accident ignition i.e. auto ignition temperature should be relatively high and should be insensitive to impacts.
7. Should have high physical strength properties – tensile, compressive and shear strength modulus of elasticity and elongation and adhesive qualities.
8. The coefficient of thermal expansion should match with that of the chamber material – this will minimize relative motion between the chamber and the thermal stresses within the stored grain.
9. Should be chemically inert with chamber material, during storage and operation.
10. The propellant should lend itself readily to production and have desirable fabrication properties – adequate fluidly during casting, easy control of chemical processes during curing and minimum shrinkage after casting or mounding.
11. The propellant's performance properties and fabrication technique should be relatively insensitive to impurities or small processing variations to simplify its production and inspection and reduce its cost.
12. The physical properties and combustion characteristics should be predictable and should not be affected appreciably over a wide range of storage and operating temperature i.e., the temperature sensitivity should be low.
13. The exhaust gases should be smokes to avoid deposits of smoke particles at operational locations and to avoid detection in military use.
14. Should lend itself readily to bonding to the metal parts, to the application of inhibitors and should be amenable to the use of a simple igniter.
15. Exhaust should be non-luminous, non-corrosive and non-toxic.
16. The method of propellant preparation should be simple and should not require a complex chemical plant installation.
17. The grain should be opaque to radiation to prevent ignition at locations other than the burning surface.

18. Should resist erosion and have predictable and repeatable erosive burning characteristics.
19. The grain should withstand repeated temperature cycling prior to operation without physical or chemical deterioration.
20. The propellant raw materials should be cheap, safe and easy to handle and transport.

Precautions in Propellant Handling

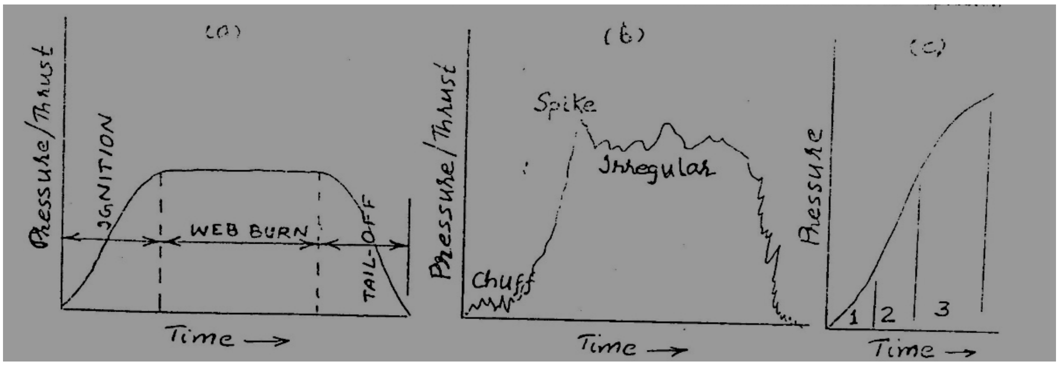
Propellants are potential explosives and require precautions during fabrication, storage and handling. The hazards pertain to (a) individual ingredients of the propellant (b) propellant undergoing mixing and other manufacturing processes (c) completed solid rocket motors and (d) the exhaust gases. Combustion explosion or denotation can occur depending on the materials and their environment initiation of energy release can be from the discharge of electricity, friction, hot embers or particles, or sudden impact.

Rigid safety precautions must be observed during solid rocket manufacture to minimise potential hazards of explosion or fire. This includes such items as a limit to the number of persons allowed at any one time in hazardous area requirements for spark-proof tools and shoes, prohibition against use of matches, cigarettes or lighters, use of remote controls and handling mechanisms, remote observation using television, close automatic watching of hazardous operations (e.g. mixing or pouring) and physical separation of buildings with potentially dangerous unit operations.

The toxicity, both dermatological and respiratory of some propellant ingredients necessitates specific safeguards and training in handling and strong the ingredients and in processing the propellants. Also the toxicity problem with HCI gas in the exhaust requires protection of operating personnel and people in nearby communities during motor firing.

IGNITION AND COMBUSTION IN SPR MOTORS

The different regime of operation of a SPR motor is shown in figure (a). They comprise of the initial ignition (transient phase) the web burn or the steady regime of burning and the final tail-off phase.



Very often there are certain anomalies observed during these three regimes. They are shown in figure (b). These abnormalities could be spiking or chuffing during ignition, irregular or oscillatory combustion or some undesirable and prolonged tail-off behavior. There could even be an explosion

IGNITION

Phase 1: Ignition time lag (Ignition decay)

The period from the moment the igniter receives a signal until the first bit of grain surface until the first bit of grain surface burns.

Phase 2: Flame spreading interval

The time from the first ignition of the grain surface until the complete grain burning area has been ignited.

Phase 3: Chamber filling interval

The time for completing the chamber filling process and for reaching equilibrium flow.

The satisfactory attainment of equilibrium chamber pressure and gas flow depends on

(a) characteristics of the igniter and gas issuing from the igniter

(b) motor propellant ignitability

(c) heat transfer characteristics between igniter gas and grain surface

(d) grain flame spreading rate

(e) the dynamics of filling the motor free volume with gas.

IGNITERS

Functions

(i) should have a very small ignition delay

(ii) should raise the temperature of the grain above its ignition temperature by releasing suffici energy

(iii) should generate enough volume of gas to build up sufficient pressure for the smooth burning the grain.

(iv) it should be stable over long periods.

(v) Should produce minimum combustion debris.

It is difficult to design an igniter, which satisfies all the above points.

Two types of ignites are (i) Pyrotechnic and (ii) Pyrogen

Pyrotechnic igniter

The figure shows typical pyrotechnic igniter (the pellet-basket design). The ignition is accomplished by stages.

Stage 1: On receipt of an electrical signal the igniter releases the energy of a small amount sensitive powered pyrotechnic housed within the initiator. This is commonly called the squib or primer charge. Squib generally contains 74% KNO3, 15.6% charcoal and 10.4% sulphur.

Stage 2: The booster charge is ignited by the heat released from the squib.

Stage 3: Finally the main charge is ignited. The main charge consists of pellets containing 2 boron, 71% potassium and 5% binder. Typical binders are epoxy resins, graphite, vegetable nitrocellulose and polyisobuiadien. The initiator and the booster charges are such that they are easy ignite and are self sustaining at low pressure.

In pyrotechnic igniters the heat transfer to the grain is by radiation.

Pyrogen igniter (see FIGURE)

A pyrogen igniter is basically a small rocket motor, which is used to ignite a large rocket motor. The pyrogen is not designed to produce thrust. The initiator and the booster charges are very similar to that in the pyrotechnic igniters. The reaction products of the main charge impinge upon the surface of the rocket motor grain causing ignition. The heat transfer form the pyrogen to the motor grain is largely convective with the hot gases contacting the grain surface.

Liquid propellants

These are classified as (i) Bipropellants, Monopropellants (ii) Hypergolic, Nonhypergolic (iii) Earth storable, Cryogenic.

Monopropellants are those, which either have fuel and oxidiser elements in themselves or are structurally so configured that on initiation by means of a catalyst or ignition energy, they decompose exothermally into gaseous products. Monopropellants, which decompose when ignited, are called 'monergois'.

Eg.:- Nitroglycerine $C_3H_5(ONO_2)_3$

Picric acid $C_6H_2(NO_2)_3CH$

Nitromethane CH_3NO_2

Monopropellants which decompose in the presence of a solid or liquid catalyst are called 'katergols'.

Eg.:- Hydrogen peroxide (H_2O_2) in the presence of silver.

Hydrazine (N2H4) in the presence of iridium.

In contrast bipropellants are those that react only when two propellant elements 'fuel' and 'oxidiser'vaporise mix and burn. Such propellants can be either hypergolic or non- hypergolic. Hypergolic combinations (hypergols) are those in which the liquid fuel and oxidiser on contact ignite release energy and burn away into hot gases. As such propulsion system based on such propellant combinations.

Eg.:- RFNA (Red fuming nitric acid)- Aniline.

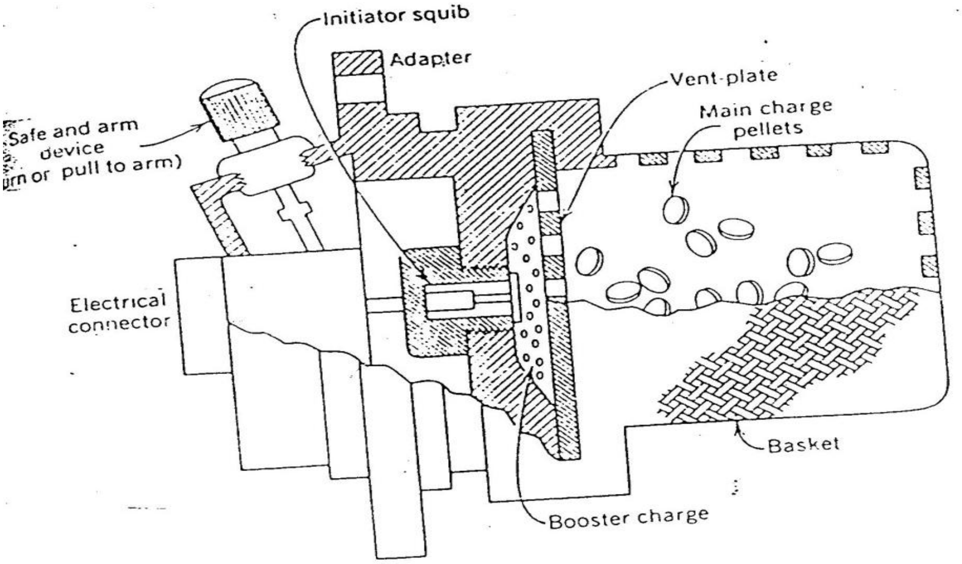
WFNA (White fuming nitric acid)- Aniline

RFNA- UDMH (Unsymmetrical Di Methyl Hydrazine)

H_2O_2-N_2H_4

N_2O_4 (Aerozine)-N_2H_4

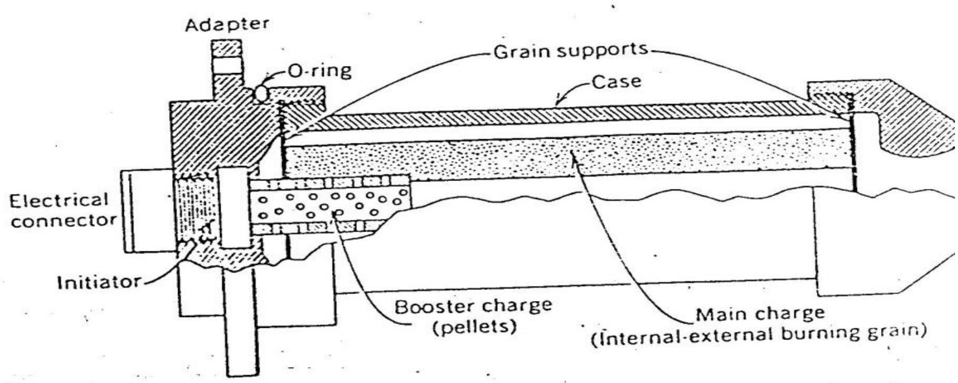
Non-hypergolic propellant combinations (Diergols) need of course, an ignition system. Eg.:- LOX (Liquid oxygen) – Ethanol



Simplified typical pyrotechnic-type igniter.

Formulations of Typical Igniter Compounds for Pyrotechnic Igniters

Type	Fuel	Fuel Percentage	Oxidizer	Oxidizer Percentage
1	Aluminum	35	$KClO_4$	65
2	Boron	30	$KClO_4$	70
3	Boron	20	KNO_3	80
4	Magnesium	60	Teflon	40



Typical pyrogen (rocket-type) igniter.

Formulations of Typical Igniter Compounds for Pyrotechnic Igniters

	Fuel	Fuel percentage	Oxidizer	Oxidizer percentage
1.	Aluminium	35	$KClO_4$	65
2.	Boron	30	$KClO_4$	70
3.	Boron	20	KNO_3	80
4.	Magnesium	60	Teflon	40

LOX – RPI (Kerosene)

LOX – LH_2 (Liquid hydrogen)

WFNA – Kerosene

The difference between cryogenic and earth storable propellant arises depending on the fact as to whether the propellant can be stored room temperatures without any boil-off or not. Cryogenic combinations need extensive insulation. Eg.:- LOX (-183^{o}C)- LH_2(-253^{o}C)

The specific impulse of liquid monopropellants is lower than of solids or liquid bipropellants. The average density of liquids is lower than of solids. The specific impulse of liquid bipropellants is generally larger than of solids.

Typical Liquid Oxidizers

1. Liquid oxygen

Boils at 90 K at atmosphere pressure. Specific gravity 1.14. Heat off vaporisation 213 Kj/Kg. It is widely used as an oxidizer and burns with a bright white-yellow flame with most hydrocarbon fuels. It has been used in combination with alcohols, jet fuels (kerosene type), gasoline and hydrogen. It is noncorrosive and nontoxic.

2. Liquid Fluorine

Boils at 53.7 K. Affords higher values of performance. High specific gravity (1.5). But toxic, corrosive and reactive.

3. Hydrogen peroxide

It is no longer used today, primarily because of its storage stability problems.

$$H_2O_2 \rightarrow H_2O + \frac{1}{2} O_2 + \text{Heat}$$

The theoretical impulse of 90% H_2O_2 is 147 sec, when used as a monopropellant.

4. Nitric acid (HNO_3)

RFNA (Red fuming nitric acid) is the most common type. RFNA consists of concentrated nitric acid that contains 5 - 20% dissolved nitrogen dioxide. The red fumes are exceedingly annoying and poisonous. Nitric acid is highly corrosive.

5. Nitrogen tetroxide (N_2O_4)

This is a high-density yellow-brown liquid (specific gravity of 1.44). It is hypergolic with many fuels and it is easily storable. The fumes are reddish-brown and are extremely toxic. Its liquid temperature is narrow and it is easily frozen or vaporised.

Liquid Fuels

1. Hydrocarbon fuels

Petroleum derivatives such as gasoline, diesel oil and turbojet fuel. RP-1 is a specially refined petroleum product particularly suitable as a rocket fuel.

2. Liquid Hydrogen when burned with liquid fluorine or liquid oxygen, gives a high performance. It is also is an excellent regenerative coolant. Liquid hydrogen is the lightest and the coldest having a specific gravity of 0.07 and a boiling point of 20 K. The low fuel density requires very bulky fuel tanks, which necessitate very large vehicle volumes. The extremely low temperature makes the problem of choosing suitable tank and piping materials difficult.

3. Hydrazine (N_2H_4)

Hydrazine is a toxic, colourless liquid with a high freezing point (274.3 K). It is an excellent monopropellant when decomposed by a suitable catalyst such as iridium. As a monopropellant it is used in gas generators or in space engine attitude control rockets. Hydrazine reacts with many materials. Compatible materials include stainless steels, nickel and 1100 and 3003 series of aluminium, Iron, copper and its alloys, monel, magnesium, zinc and some types of aluminium alloys must be avoided.

4. Unsymmetrical DiMethyl Hydrazine (UDMH) $[(CH_3)_2 NNH_2$

A derivative of hydrazine. Freezing 215.9 K and boiling point is 336.5 K. It is toxic, but least toxic among hydrazine, MMH and UDMH. UDMH is now made in India in fairly large quantities.

5. Monomethyl Hydrazine (MMH) $[(CH_3 NHNH_2)]$

MMI has been used extensively as a fuel in rocket engines, particularly in small attitude control engines, usually with as the oxidiser. MMH is the most toxic among all hydrazines.

Propellant Selection and Desirable Properties

The selection of the propellant combination is a compromise of various factors such as those listed below.

1. Economic Factors

Availability and low cost (Impulse/unit cost must be as large as possible).

In military applications, consideration has to be given to logistics of production, supply and other possible military uses. The production process should be simple, requiring only ordinary chemical equipment and available raw materials.

2. Performance of propellants

For high content of chemical energy per unit of propellants mixture is desirable because it permits a high chamber temperature. A low molecular mass of the product gases is also desirable.

3. Common physical hazards

Corrosibility should be low. Properties like toxicity, fire and explosion hazards must be minimum.

4. Desirable physical properties

(i) Freezing point should be low. This permits operation of rockets in cold weather.

(ii) High specific gravity - In order to accommodate a large weight of propellants in a given vehicle tank space, a dense propellant is required. It permits a small vehicle construction and consequently, a relatively low structural weight and low structural weight and low aerodynamic drag.

(iii) Stability - No determination with long term (over 10 years) storage and minimal reaction with the atmosphere are desirable. Should have a negligible chemical reaction with piping, tank walls, valve seats and gasket materials even at relatively high ambient temperatures.

(iv) Heat transfer properties - High specific heat, high thermal conductivity and a high boiling or decomposition temperature are desirable for propellants that are used for thrust chamber cooling.

(v) Pumping properties - A low vapour pressure permits easier handling of the propellants and a more effective pump design in applications where the propellant is pumped. If the viscosity of the propellant is too high then pumping becomes difficult.

(vi) Temperature variation - The temperature variation of the physical properties of the liquid propellant should small.

5. Ignition, Combustion and Flame properties

All rocket propellant should be readily ignitable and have a small ignition time delay in order to reduce the explosion hazard during starting.

Propellants should burn smoothly without combustion instabilities.

Reaction rate should be fast so that its residence time is a minimum. This reduces the length of the combustion chamber and hence the weight.

Smoke formation is objectionable. Smoke and brilliantly exhaust flames are objectionable in certain military applications, because they can easily be detected. In some applications the condensed species in the exhaust gas can cause surface contamination on spacecraft windows or optical lenses and the electrons in the flame can cause undesirable interference or attenuation of communications radio signals.

Starting and Ignition of LPR

The starting of a thrust chamber has to be so controlled that a timely and even ignition of propellants is achieved and the flow and thrust are built up smoothly and quickly to their rated values.

The initial propellant flow should be less than the full flow - this prevents an excessive, accumulation of the unignited propellants in the chamber during the ignition delay and tends to assure a smooth build up of chamber pressure. If the stating propellant flow is excessive, explosion may sometime be severe enough to cause the destruction of the rocket engine.

The starting mixture ratio is different from the operating mixture ratio - during starting the injection velocity is low, the initial vaporisation and mixing of the propellants in the cold combustion chamber is incomplete and there will be local regions of local regions of lean and rich mixtures. The optimum starting mixture is therefore an average of a range of mixture ratios which are readily ignited. Mixture ratios near the stoichiometric ratio have a heat release per unit of propellant and therefore brings the chamber condition to equilibrium faster than would be possible with other mixtures. The opening mixture ratio is usually fuel rich.

Sometime a more reliable ignition is assured when one of the propellants is intentionally made to reach the chamber first. For a fuel rich starting mixture the fuel is admitted first while for a fuel lean mixture is given the lead.

Ignition

Nonhypergolic propellants need to be activated by absorbing energy prior to combustion initiation. This energy is supplied by the ignition system. Once ignition has begun the flame is self supporting. The igniter has to be located in such a manner that a satisfactory starting mixture at low initial flow is present at the time of activation. There are four different types of ignition systems, which have been tried.

Spark plug ignition

The spark plug is located in a region where initial fuel oxidiser and vapours will form an ignitable mixture. This method is attractive in small rocket engines which has to be restarted during flight. Found good for LOX-gasoline, nitromethane-oxygen and LOX - LH_2 thrust chambers.

Pyrotechnic ignition

This uses a solid propellant squib or grain of a few seconds burning duration. The solid propellant charge is electrically ignited and burns with a hot flame. This method can be used only once and thereafter the charge has to be replaced.

Pre-combustion chamber ignition

A small chamber is built next to the combustion chamber connected through an orifice similar to the pre-combustion chambers for compression ignition engines. A small amount of fuel and oxidiser is injected into the pre-combustion chamber and ignited by a spark plug, catalyst or other means. The burning mixture enters the main propellant flow which is injected into the main chamber. This method also permits repeated starting. It is found successful for LOX - Gasoline and LOX Alcohol engines.

Auxiliary Fluid injection

This is a method where some liquid or gas in addition to the regular fuel and oxidiser is injected into the combustion chamber for very short periods during starting. This fluid should be hypergolic with either the fuel or the oxidiser and this produces spontaneous combustion. Eg.:- Nitric acid (HNO_3) with some organic fuel, combustion can be initiated by introducing a small quantity of aniline during starting.

The Injectors

The injection head is as important as a carburetor for a SI engine. The injector has to introduce and meter the flow to the combustion chamber and atomize and mix the propellants in such a manner that a correctly proportioned homogeneous fuel-oxidizer mixture will result which can readily be vaporized and burnt.

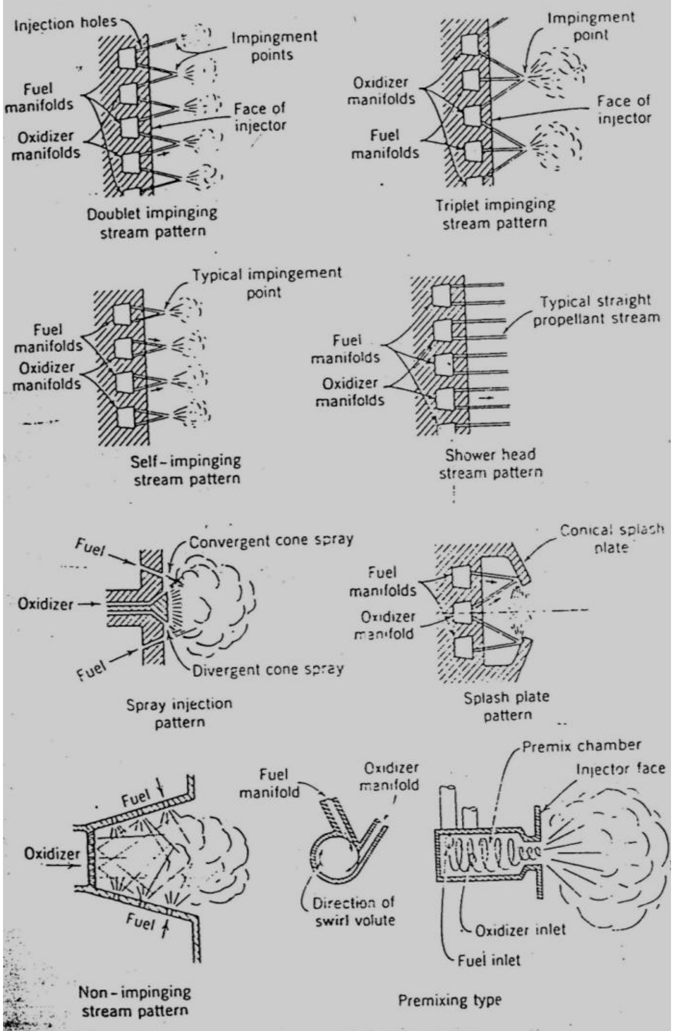
In the impinging stream type the propellants are injected through a number of separate small holes in such a manner that the fuel and oxidizer streams impinge upon each other. This will aid the atomization of the liquids into droplets and the distributor. Majority of the rocket engines use impinging stream type injector.

The non-impinging or shower head injector relies on turbulence and diffusion to achieve mixing.

In the splash plate type injectors mixing is achieved by impinging the propellant streams against a surface.

Sheet type or spray type injectors give cylindrical, conical or other types of spray sheets which interest and thereby promote mixing.

In premixing type the mixing of the liquid propellants in achieved before they are introduced into the combustion chamber.



Schematic diagrams of several injector types.

The following steps take place in the combustion chamber of a liquid propellant rocket engine.

ELECTRICAL ROCKETS

Electrical rocket propulsion devices use electrical energy form a separate energy source for ejecting propellant mass. Flight in space, especially that lasting months or years, increase the importance of two propulsion design criteria, namely, long engine life with high reliability and low propellant consumption rates or high specific impulse. Electric propulsion offers both.

The basic subsystems of a typical electrical propulsion system are (a) a power source (usually solar, nuclear or chemical) with its auxiliaries such as pumps, panels, radiators and controls (b) a conversion device to convert this energy into an electrical form at the proper voltage, frequency and current suitable for propulsion. (C) a propellant system for storing metering and delivering a propellant. (D) one or more thrusters to convert the electrical energy into kinetic energy of the exhaust.

Three fundamental types of electrical rockets are

(1) Electrothermal - Propellant is heated electrically and is thermodynamically expanded and the heated gas in accelerated to supersonic speeds through a nozzle as in a chemical rocket.

(2) Electrostatic - Acceleration is achieved by the interaction of electrostatic fields on charged propellant particles, such as ions and colloids.

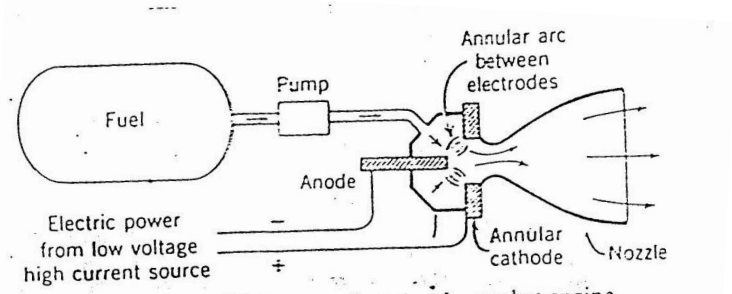
(3) Electromagnetic - Acceleration is achieved by the interaction of electric magnetic fields on propellant plasma. A plasma is a high temperature electrically neutral gas containing electrons, ions and neutral molecular species.

The application for electrical propulsion systems are as a means (1) for changing the orbits or overcoming perturbations of earth satellites (2) for correcting space-light trajectories (3) for space vehicle attitude control (4) for achieving interplanetary transfers or solar system escape.

A schematic diagram of an arc heating rocket (electrothermal type) engine is shown in figure (ref FIGURE). Electrical energy is transferred into heat in an arc struck between two electrodes and, thus the temperature of the working fluid is raised as it passes through a nozzle and thus is accelerated and ejected at high velocities (7500-20000 m/s).

NUCLEAR ROCKETS

Nuclear energy can be utilised in rocket propulsion in the form of fission energy release, radioactive isotope decay and fusion energy release. Nuclear rockets are broadly classified according to these types of energy release. Of the three forms of nuclear energy, radioactive decay is the one suited to low thrust rockets (5N or less) and is most advanced in its development. Both fission and fusion when applied to rocket propulsion, inherently result in high thrust devices, typically in the 250-2500 KN thrust range.



Schematic diagram of arc-heating rocket engine.

(See FIGURE)

Figure schematically describes the essential components and arrangement of a solid-core nuclear rocket engine. The engine contains the nuclear fission reactor supported within the pressure shell, the turbo pump propellant feed system, hydrogen cooled exhaust nozzle and the necessary reactor power and propellant flow controls. In the fission reactor heat is generated by the fission of uranium. This heat is subsequently transferred to the working fluid. The high temperatures of the reactor and the radiation effects on materials and on humans present special problems.

SOLAR ROCKETS

Several methods exist for harnessing solar energy to propel spacecraft. The use of solar cells to generate electricity for electric rocket engines is the most highly developed. Other methods include solar heating of the propellants (work fluid - hydrogen) as shown in figure (ref FIGURE) and use of solar sail.

JET PROPULSION ENGINES

Theory of jet propulsion

Jet propulsion is based on Newton's second and third low of motion. Newton's second law states that 'the range of momentum in any direction is proportional to the force acting in that direction'. Newton's third low states that for every action there is an equal and opposite reaction.

In propulsion momentum is imparted to a mass of fluid in such a manner that the reaction of the imparted momentum furnishes a propulsive force. The jet aircraft draws in air and expels it to the rear at a markedly increased velocity; the rocket greatly changes the velocity of its fuel which it ejects rearward in the form of products of combustion. In each case the action of accelerating the mass of fluid in a given direction created a reaction in the opposite direction in the form of a propulsive force. The magnitude of this propulsive force is defined as thrust.

Types

The jet propulsion engines are classified basically as to their method of operation. The two main categories of jet propulsion engines are the atmospheric jet engines and the rockets. The atmospheric jet engines require oxygen from the atmospheric air for the combustion of fuel. As a result, their performance depends to a great degree on the forward speed of the engine and upon the atmospheric pressure and temperature.

The rocket engine differs from the atmospheric jet engines in that the entire mass of jet is generated from the propellants carried within the engine, i.e., the rocket engine carries its own oxidant for the combustion of the fuel and is therefore, independent of the atmospheric air. The performance of this type of power plant is independent of the forward speed and affected to a maximum of about 10% by changes in altitude.

Air Breathing Engines

Air breathing engines can further be classified as follows:

1. Reciprocating engines (Air screw)

2. Gas Turbine engines (i) Turbojet (ii) Turbojet with after burner (also known as turbo ramjet, turbojet with tail pipe burning and turbojet with reheater) (iii) Turboprop (also known as propjet).

3. Athodyds (Aero Thermodynamics Ducts)

(i) steady combustion system, continuous air flow – Ramjet (also known as Lorin tube)(ii) Intermittent combustion system, intermittent air flow – Pulse jet (also known as aero pulse, resojet, Schmidt tube and intermittent jet).

The reciprocating engine develops its thrust by accelerating the air with the help of a propeller driven by it, the exhaust of engine imparting almost negligible amount of thrust to that developed by the propeller.

The turbojet, turbojet with afterburner and turboprop are modified simple open cycle gas turbine engines. The turbojet engine consists of an open cycle gas turbine engine (compressor, combustion chamber and turbine) with an entrance air diffuser added in front of the compressor and an exit nozzle added aft of the turbine. The turbojet with afterburner is a turbojet engine with a reheater added to the engine so the extended tail pipe acts as a combustion chamber. The turboprop is a turbojet engine with extra turbine stages, a reduction gear train and a propeller added to the engine. Approximately 80 to 905 of the thrust of the turboprop is produced by acceleration of the air outside the engine by the propeller and about 10 to 20% of the thrust is produced by the jet exit of the exhaust gases. The ramjet and the pulsejet are athodyds, i.e., a straight duct type of jet engine without compressor and turbine wheels.

Rocket Engines

The necessary energy and momentum which must be imparted to a propellant as it is expelled from the engine to produce a thrust can be given in many ways. Chemical, nuclear or solar energy can be used and the momentum can be imparted by electrostatic or electromagnetic force.

Chemical rockets depend up on the burning of the propellant inside the combustion chamber and expanding it through a nozzle to obtain thrust. The propellant may be solid, liquid, gas or hybrid.

The vast store of atomic energy is utilized incase of nuclear propulsion. Radioactive decay or Fission or Fusion can be used to increase the energy of the propellant.

In electrical rockets electrical energy from a separate energy source is used and the propellant is accelerated by expanding in a nozzle or by electrostatic or electromagnetic forces.

In solar rockets solar energy is used to propel spacecraft.

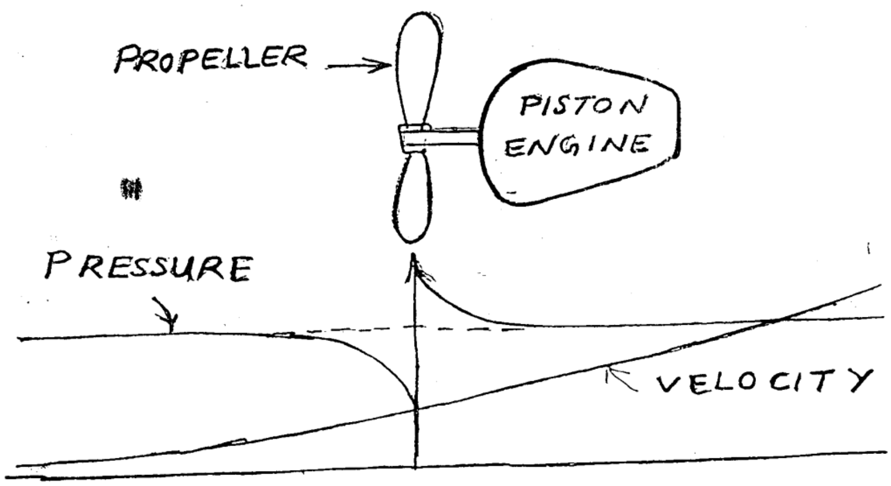
Differentiate between air breathing engines and rocket engines?

AIR BREATHING ENGINES	ROCKET ENGINES
Combustion takes place by using atmospheric air.	Combustion takes place by using its own oxygen supply.
These engines cannot be used at very high altitudes due to deficiency of air.	Rocket engines can be used at very high altitudes. That‘s why it is used in space crafts and for satellite launching
It do not require any oxygen storage system	Requires oxygen storage system.
Design is not much complex.	Design is very complex.
Air breathing engines can be either reciprocating or rotary type.	Rocket engines are of rotary type.
Air breathing engines can be used for domestic applications.	Rocket engines are used for scientific researches mainly in outer space
Air breathing engines are not much bulkier.	Rocket engines are really bulkier.

AIR SCREW

In an airscrew the source of power is a reciprocating internal combustion engine which drives a propeller connected to it. The propeller displaces rewards a large mass of air, accelerating it in the process (Figure 1). Due to this acceleration of the fluid a propulsive force is produced which drives the aircraft.

Figure 1·



Nearly all the earlier aircrafts used reciprocating engines as the source of energy to drive the propeller. The use of reciprocating engines is continuously on the decline because its development has reached a stage of near saturation. Present day aircrafts demand high flight speeds, long distance travels and high load carrying capacities. A power output more than 5000 hp. is difficult to obtain without modifications in the present reciprocating engine plant. The output can be increased by increasing the cylinder size, installing large number of cylinders or by running the engine at high speeds. Unfortunately all those methods of raising the output of the engine increase the engine size, frontal area of the aircraft, complexity and cost of the plant. The drag of the plant will also increase to critical values with increase in engine size.

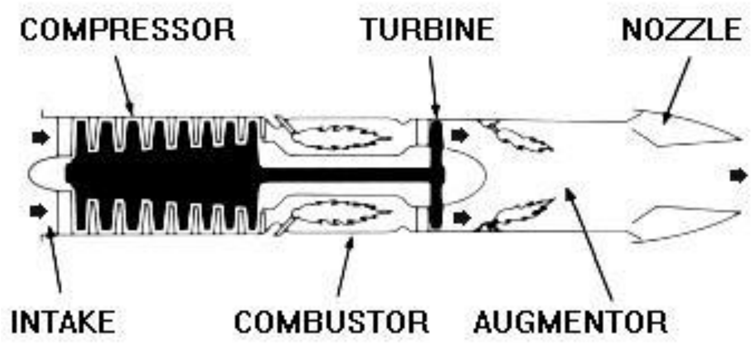
The speed of airscrew is limited to a range of about 700 km/h. The propeller loses its effectiveness at higher speeds due to separation of flow and shock waves as the air velocity approaches the sonic velocity. At lower speeds the propulsive efficiency of the propeller is about 95%.

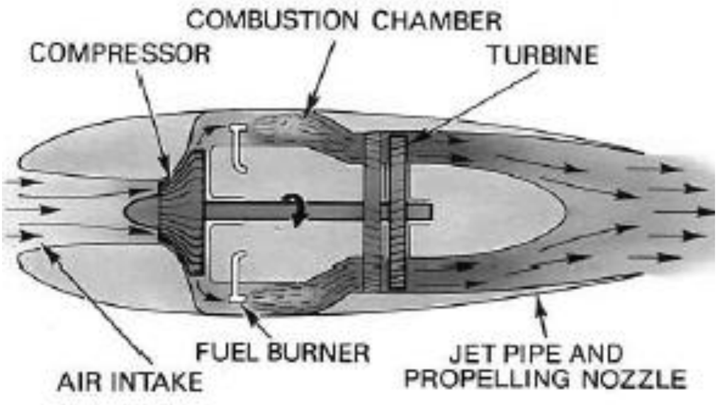
For small aircrafts flying at velocities less than about 500 to 650 km/h reciprocating engine has an enviable position due to its excellent fuel economy and good take-off characteristics. However due to comparatively large drop in power with altitude operation and the need of using high octane fuels, along with the difficult cooling and lubrication problems, high weight/power ratio, and larger frontal area of such engines these are being replaced by turbojets in higher speed ranges.

The Turbojet Engine

The turbojet engine consists of diffuser which shows down the entrance air and thereby compresses it, a slows down the entrance air and thereby compresses it, a simple open cycle gas turbine and an exist gas into kinetic energy. The increased velocity, of air thereby produces thrust.

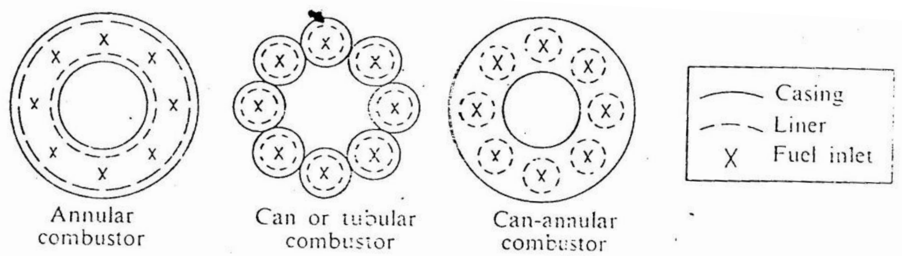
Figure 2 shows the basic arrangement of the diffuser, compressor, combustion chamber, turbine and the exhaust nozzle of a turbojet engine. Of the total pressure rise of air, a part is obtained by the rain compression in the diffuser and rest in the compressor. The diffuser converts kinetic energy of the air into pressure energy. In the ideal diffuser, the air is diffused isentopically down to zero velocity. In the actual diffuser the process is irreversible adiabatic and the air leaves the diffuser at a velocity between 60 and 120 m/s.





The composer used in a turbojet can be either centrifugal type or axial flow type. The centrifugal compressor gives a pressure ratio of about 4:1 to 5:1 in a single stage and usually a double-sided rotor is used. The turbojet using centrifugal compressor has a short and sturdy appearance. The advantages of centrifugal compressor are high durability, ease of manufacture and low cost and good operation under adverse conditions such as icing and when sand and small foreign particles are inhaled in the inlet duct. The primary disadvantage is the lack of straight-through airflow. Air leaves compressor in radial direction and ducting with the attendant pressure losses is necessary to change the direction. The axial flow is more efficient than the centrifugal type and gives the turbojet a long slim, streamlined appearance. The engine diameter is reduced which results in low aircraft drag. A multistage axial flow compressor can develop a pressure ratio as high as 6:1 or more. The air handled by it is more than that handled by a centrifugal compressor of the same diameter. A variation of the axial compressor, the twin-spool (dual spool, split spool or coaxial) compressor has two or more sections, each revolving at or near the optimum speed for its pressure ratio and volume of air. A very high-pressure ratio of about 9:1 to 13:1 is obtained. The use of high-pressure ratio gives very good specific fuel consumption and is necessary for thrust ratings in the region of 50000 N or greater.

In the combustion chamber heat is added to the compressed air nearly at constant pressure. The three types being ‘can’, ‘annular’ and ‘can-annular’ (ref.fig.3). In the can type individual burners, or cans, are mounted in a circle around the engine axis with each one receiving air through its own cylindrical shroud. One of the main disadvantages of can type burners is that they do not make the best use of available space and this results in a large diameter engine. On the other hand, the burners are individually removable for inspection and air-fuel patterns are easier to control than in annular designs. The annular burner is essentially a single chamber made of concentric cylinders mounted co-axially about the engine axis. This arrangement makes more complete use of available space, has low pressure loss, fits well with the axial compressor and turbine and form a technical viewpoint has the highest efficiency, but has a disadvantage in that structural problems may arise due to the large diameter, thin-wall cylinder required with this type of chamber. The problem is more severe for larger engines. There is also some disadvantage in that the entire combustor must be removable from the engine for inspection and repairs. The can-annular design also makes good use of available space, but employs a number of individually replaceable cylindrical inner liners that receive air through a common annular housing for good control of fuel and air flow patterns. The can-annular arrangement has the added advantage of greater structural stability and lower pressure loss than that of the can type.



Combustion chamber, longitudinal and cross sections.

Figure 3

The heated air then expands through the turbine thereby increasing its velocity while losing pressure. The turbine extracts enough energy to drive the compressor and the necessary auxiliary equipments. Turbines of the impulse, reaction and a combination of both types are used. In general, it may be stated that those engines of relatively low thrust and simple design employ the impulse type, while those of large thrust employ the reaction and combination types.

The hot gas is then expended through the exit nozzle and the energy of the hot gas is converted into as much kinetic energy as is possible. This change in velocity of the air passing through the engine multiplied by the mass flow of the air is the change of momentum, which produces thrust. The nozzle can be a fixed jet or a variable area nozzle. The variable area nozzle permits the turbojet to operate at maximum efficiency over a wide range of power output. Clamshell, Finger or Iris, Centre plug with movable shroud, annular ring, annular ring with movable shroud are the various types of variable area nozzle for turbojet engines. The advantage of variable area nozzle is the increased cost, weight and complexity of the exhaust system.

The needs and demands being fulfilled by the turbojet engine are

1. Low specific weight – ¼ to ½ of the reciprocating engine
2. Relative simplicity – no unbalanced forces or reciprocating engine
3. Small frontal area, reduced air cooling problem- less than ¼ th the frontal area of the reciprocating engine giving a large decrease in nacelle drag and consequently giving a greater available excess thrust or power, particularly at high speeds.
4. Not restricted in power output - engines can be built with greatly increased power output over that of the reciprocating engine without the accompanying disadvantages.
5. Higher speeds can be obtained – not restricted by a propeller to speeds below 800 km/h.

Turboprop Engine (Propeller turbine, turbo-propeller, prop jet, turbo-prop)

For relatively high take-off thrust or for low-speed cruise applications, turboprop engines are employed to accelerate a secondary propellant stream, which is much larger than the primary flow through the engine. The relatively low work input per unit mass of secondary air can be adequately transmitted by a propeller. Though a ducted fan could also be used for this purpose, a propeller is generally lighter compared to ducted fan could also be used for this purpose, a propeller is generally lighter compared to ducted fan engine and with variable pitch, it is capable of a wider range of satisfactory performance.

In general, the turbine section of a turboprop engine is very similar to that of a turbojet engine. The main difference is the design and arrangement of the turbines. In the turbojet engine the turbine is designed to extract only enough power from the high velocity gases to drive the compressor, leaving the exhaust gases with sufficient velocity to produce the thrust required of the engine. The turbine of the turboprop engine extracts enough power from the gases to drive both the compressor and the propeller. Only a small amount of power is left as thrust. Usually a turboprop engine has two or more turbine wheels. Each wheel takes additional power from the jet stream, with the result that the velocity of the jet is decreased substantially.

Figure 6 shows a schematic diagram of a turboprop engine. The air enters the diffuser as in a turbojet and is compressed in a compressor before passing to the combustion chamber. The compressor in the turboprop is essentially an axial flow compressor. The products of combustion expand in a two-stage or multistage turbine. One stage of the turbine drives the compressor and the other drives the propeller. Thus the turbine expansion is used to drive both compressor as well as propeller and less energy is available for expansion in the nozzle. Due to lower speeds of propeller a reduction gear is necessary between turbine and the propeller. About 80 to 90% of the available energy in exhaust is extracted by the turbine while rest, about 10 to 20%, contributes the thrust by increasing the exhaust jet velocity.

Total thrust = jet thrust + propeller thrust

Turboprop engines combine in them the high take-off thrust and good propeller efficiency of the propeller engines at speeds lower than 800 km/h and the small weight, lower frontal area and reduced vibration and noise of the pure turbojet engine.

Its operational range is between that of the propeller engines and turbojets though it can operate in any speed up to 800 km/h.

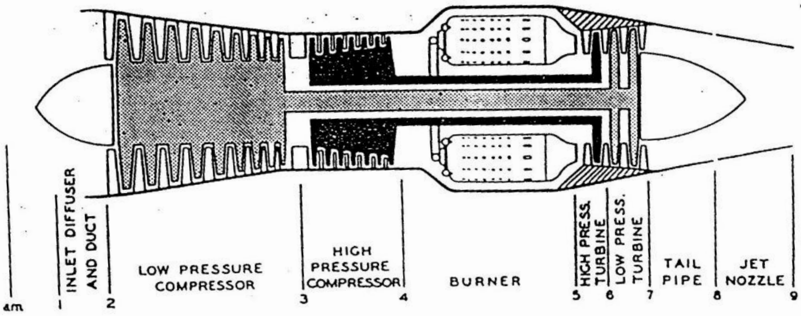


Figure 4 Arrangement of a split-compressor, axial-flow engine.

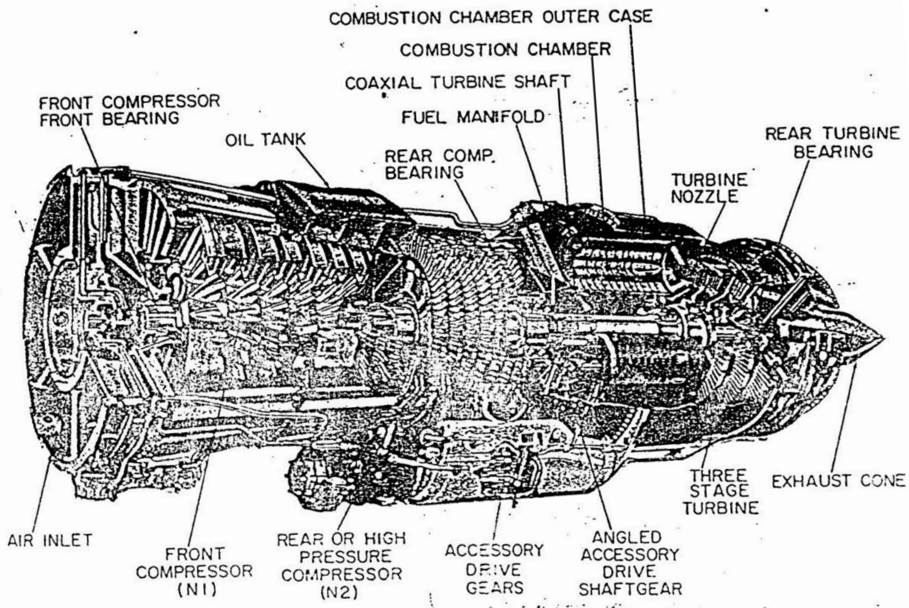


Figure 5 Pratt & Whitney J57 turbojet engine.

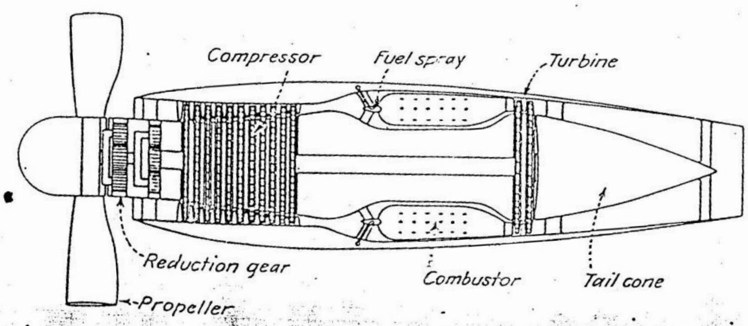
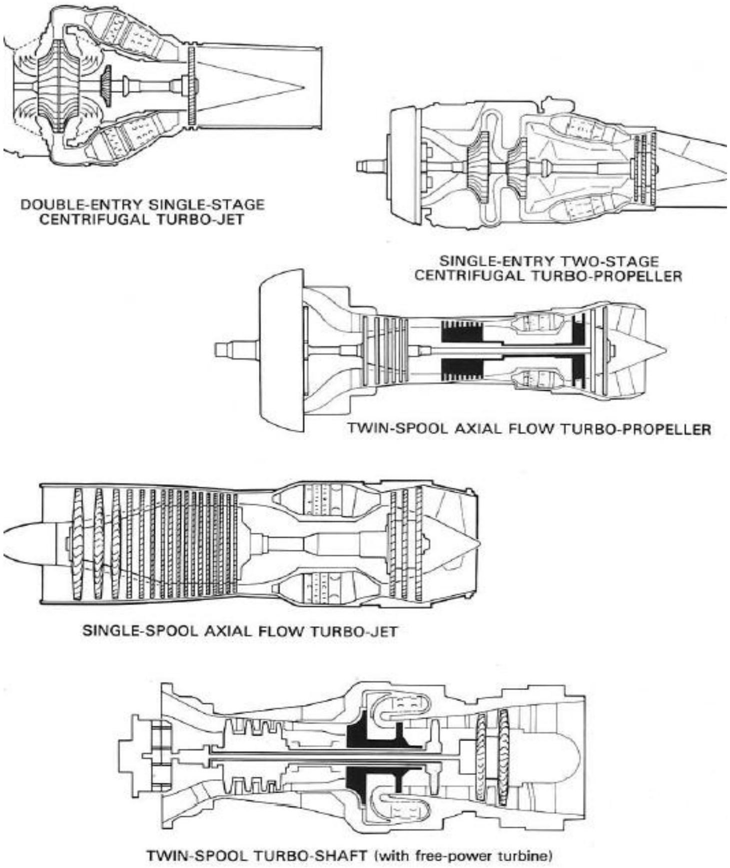
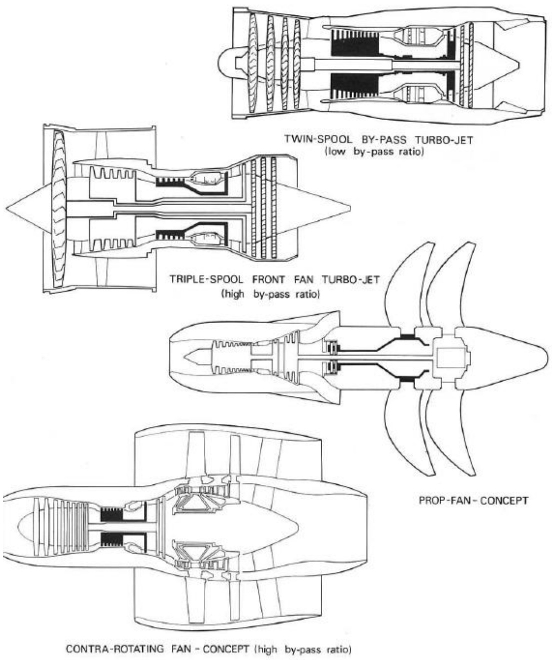


Figure 6 Simplified drawing of a turboprop engine.





The power developed by the turboprop remains almost same at high altitudes and high speeds as that under sea-level and take-off conditions because as speed increases ram effect also increases. The specific fuel consumption increases with increase in speed and altitude. The thrust developed is high at take-off and reduces at increased speed.

Advantages

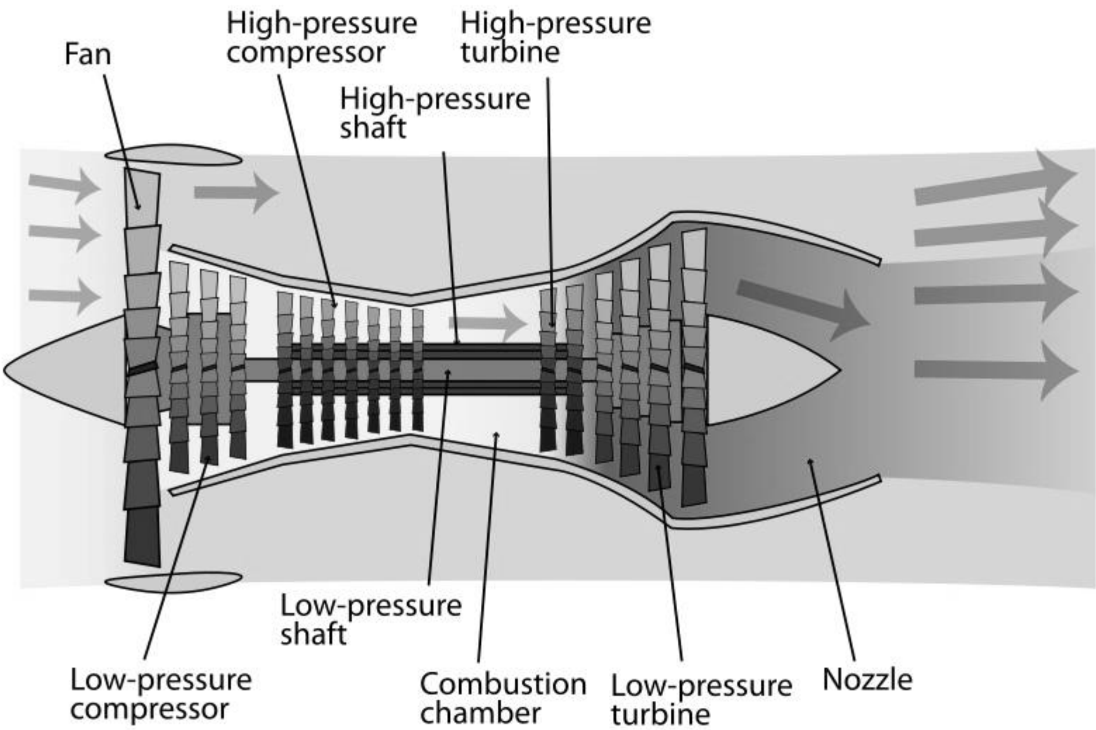
1. Turboprop engines have a higher thrust at take-off and better fuel economy.
2. The frontal area is less than air screw so that drag is reduced.
3. The turboprop can operate economically over a wide range of speeds ranging from low speeds, where pure jet engine is uneconomical, to speeds of about 800 km/h where the propeller engine efficiency is low.
4. It is easy to maintain and has lower vibrations and noise.
5. The power output is not limited as in the case of propeller engines (air screw).
6. The multicast arrangement allows a great flexibility of operation over a wide range of speeds.

Disadvantages

1. The main disadvantage is that at high speeds due to shocks and flow separation, the propeller efficiency decreases rapidly, thereby, putting up a maximum speed limit on the engine.
2. It requires a reduction gear which increases the cost and also consumes certain energy developed by the turbine in addition to requiring more space.

The Turbofan Engine

The turboprop is limited to mach number of about 0.7 because of the sharp decrease in propeller efficiency encountered above that mach number. However, the turboprop concept of increasing mass flow rate without producing an excessive increment in exhaust velocity is valid at any mach number and the use of a ducted fan combined with a jet turbine provides more economical operation at mach numbers close to unity than does the simple jet turbine. If a duct or shroud is placed around a jet engine and air is pumped through the annular passage by means of one or more sets of compressor blades, the resulting engine is called a turbofan, and is capable of producing (under proper conditions) somewhat better thrust specific fuel consumption characteristics than the turbojet itself. Basically, the air passing through the fan bypasses the combustion process but has energy added to it by the compressor fan, so that a sizable mass flow can be shunted through the fan. The air which bypasses the combustion process leaves the engine with a lower amount of internal energy and a lower exhaust speed than the jet exhaust. Yet, the thrust is not decreased since the turbofan can pump more air per unit time than a conventional jet at subsonic speeds. Accordingly, the average exhaust velocity of the turbofan (averaging the turbine flow and the bypass flow) can be made smaller at a given flight speed than that of a comparable turbojet and greater efficiency can be obtained. In turbofan engine the fan cannot be designed for all compressor ratios which is efficient at all mach numbers, thus, the turbofan is efficient over a rather limited range of speeds. Within this speed range, however, its improved cruise economy makes it a desirable unit for jet transport aircraft.



The turbofan engine has a duct enclosed fan mounted at the front or rear of the engine and driven either mechanically geared down or at the same speed as the compressor, or by an independent turbine located to the rear of the compressor drive turbine (Ref. Figure 7). There are two methods of handling the fan air. Either the fan can exit separately from the primary engine air, or it can be ducted back to mix with the primary engine's air at the rear. If the fan air is ducted to the rear, the total fan pressure must be higher than the static pressure in the primary engine's exhaust, or air will not flow. Similarly, the static fan discharge pressure must be less than the total pressure the primary engine's exhaust, or the turbine will not be able to extract the energy required to drive the compressor and fan. By closing down the area of flow of the fan duct, the static pressure can be reduced and the dynamic pressure is increased.

The efficiency of the fan engine is increased over that of the pure jet by converting more of the fuel energy into pressure energy rather than the kinetic energy of a high velocity exhaust gas stream. The fan produces additional force or thrust without increasing fuel flow. As in the turboprop primary engine exhaust gas velocities and pressures are low because of the extra turbine stages needed to drive the fan, and as a result this makes the turbofan engine much quieter. One fundamental difference between the turbofan and the turboprop engine is that the air flow through the fan is controlled by design so that the air velocity relative to the fan blades is unaffected by the aircraft's speed. This eliminates the loss in operational efficiency at high air speeds which limits the maximum air speed of propeller driven aircraft.

Fan engines show a definite superiority over the pure jet engines at speeds below Mach 1. The increased frontal area of the fan presents a problem for high-speed aircraft which, of course require small frontal areas. At high speeds air can be offset at least partially by burning fuel in the fan discharge air. This would expand the gas, and in order to keep the fan discharge air at the same pressure, the area of the fan jet nozzle is increased. This action results in an increase in gross thrust due to an increase in pressure times an area (PA), and an increase in gross thrust specific fuel consumption.

Turbo shaft Engine

A turboshaft engine might be defined as a gas turbine engine designed to produce only shaft power. Such engines find application in helicopters, where their light weight and small size compared with piston engines render them attractive. Turboprop engines are similar to turboprop engines, expect that the hot gases are expanded to a lower pressure in the turbine, thus providing little exhaust velocity.

Comparison of turbojet, Turboprop and Turbofan engines (Figure 8)

Turbojet

1. Low thrust at low forward speeds

2. Relatively high TSFC at low altitudes and low speeds. This disadvantage decreases as altitude and speed increase.

3. Long take-off road is required.

4. Small frontal area results in reduced ground clearance problem.

5. Lightest specific weight.

6. Ability to take advantage of high rams pressure ratios.

Suitable for high speed, high altitude long distance flights.

Turboprop

1. High propulsive efficiency at low speeds which falls off rapidly as speed increases this results in shorter take-off roads. The engine is able to develop very high thrust at low speeds because the propeller can accelerate large quantities of air at zero forward velocity of the airplane.

2. More complicated and heavier than the turbojet engine.

3. Lowest TSFC.

4. Large frontal area of propeller and engine combination necessitates longer landing gears for lowing airplanes.

5. Efficient reserve thrust possible.

Turboprop engines are superior for lifting heavy loads off short and medium runways

Turbofan

1. Increased thrust at forward speeds similar to a turboprop results in a relatively short take-off. But unlike the turboprop the turbofan does not decrease in thrust with increasing speed. It can be used approximately up to Match 1.

2. The weight lies some where between the turbojet and turboprop.

3. Ground clearances are less that for turboprop but more than that for turbojet.

4. TSFC and specific weight fall between turboprop and turbojet. Increased operating economy in 800 – 1200 km/h. range.

5. Considerable noise level reduction over equivalent turbojet engines.

6. Superior to turbojet in ‘hot day’ performance.

7. Two thrust reverses are required if the fan air and primary engine exhaust through separate nozzles.

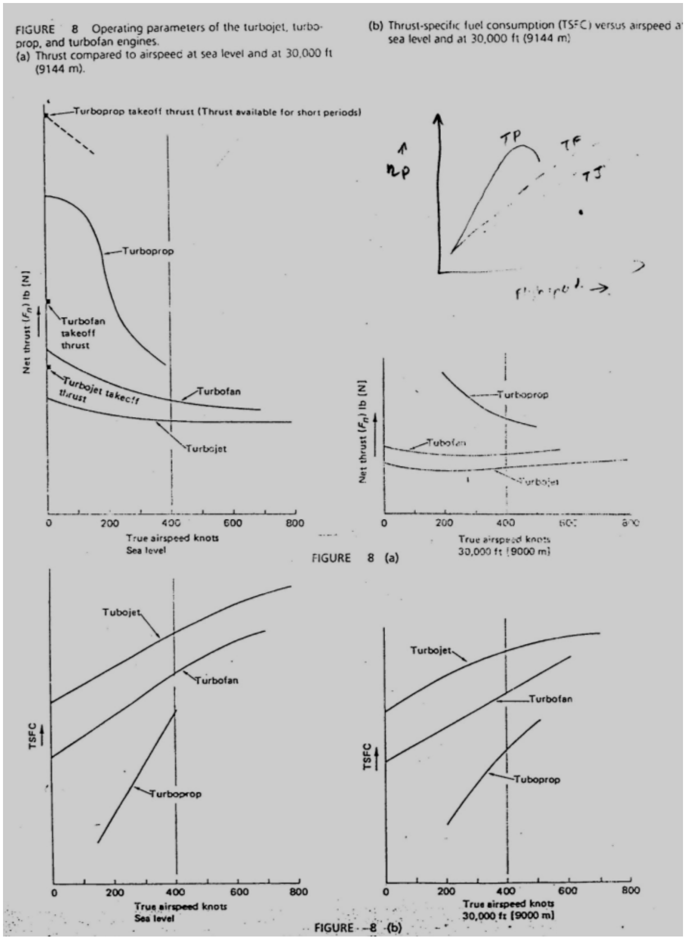


FIGURE 8 Operating parameters of the turbojet, turboprop, and turbofan engines.
(a) Thrust compared to airspeed at sea level and at 30,000 ft (9144 m).
(b) Thrust-specific fuel consumption (TSFC) versus airspeed at sea level and at 30,000 ft (9144 m)

Fan engine is suitable for long range, relatively high speed flight.

Comparison is made by assuming that the engines are equivalent as to compressor ratio and internal temperatures and that the engines are installed in equal sized aircraft best suited to the particular type of engine being used.

Advantages and Disadvantages of the Gas Turbine as Aircraft power plant

Advantages

1. Freedom from vibration-permits lighter propeller sections and mounting structure.
2. Simplicity of control.
3. No radiators or other cooling surfaces.
4. Negligible cooling air required.
5. No spark plugs required expect for starting – once combustion is established, it is self supporting.
6. No carburetors.
7. Available supply of compressed air.
8. Decreased fire hazard – less volatile fuels are used.
9. Lower specific weight.
10. Lower oil consumption.

Disadvantages

1. High specific fuel consumption at low air speeds – applies chiefly to pure jet engines have performance comparable to reciprocating engines.
2. Inefficient operation at low power levels.
3. Slow acceleration from minimum to maximum power level – this condition applies chiefly to turbojet engines. Turboprop and turbofan engines are able to accelerate quite rapidly.
4. High starting power requirements.
5. High cost manufacture.

6. Susceptibility to damage by foreign material – such material is readily drawn into the air inlet.

Basic Differences – Gas Turbines Vs. Reciprocating Engines

Aerodynamic

Advantages – similar nacelles possible; negligible cooling power required; high speed jet a more efficient propulsive means than propellers at high flight speeds.

Disadvantages – A high speed jet is less efficient propulsive means than a propeller at lower flight speeds and burning take-off. The development of turboprop and turbofan engines has made it possible to combine the advantages of the turbine engine with the efficiency of the propeller for lower speeds and take-off.

Weight

Turbine engines are considerably lighter than reciprocating engines for the same power output

Turbine engines	<0.13 N/N thrust.
Turboprop engines	<2.36 N/kW
Piston engines	<6.05

Fuel consumption

Neutral characteristics – Best SFC occurs near maximum output. (Best SFC of reciprocating engines occurs at about one-half maximum power). At a given flight speed, SFC of the turbojet engine tends to decrease with altitude. The SFC for turboprop engine is comparable to that of the reciprocating engines.

Disadvantages – Best fuel consumption is in general poorer for the turbojet engine.

Output

Neutral characteristics - Operation of the gas turbine engine at varying altitude is somewhere between that of an unsupercharged and a supercharged reciprocating engine. That is, the turbine engine is more adaptable to varying altitude than the unsupercharged engine and perhaps a little less adaptable than the supercharged reciprocating engine.

General

Advantages – Low power plant vibration, relatively constant speed over a wide range of output.

Disadvantage- High engine speed

THE RAMJET ENGINE

The ramjet engine is an air breathing engine which operates on the same principle as the turbojet engine. Its basic operating cycle is similar to that of the turbojet. It compresses the incoming air by ram pressure, adds the heat energy to velocity and produces thrust. By converting kinetic energy of the incoming air into pressure, the ramjet is able to operate without a mechanical compressor. Therefore the engine requires no moving parts and is mechanically the simplest type of jet engine which has been devised. Since it depends on the velocity of the incoming air for the needed compression, the ramjet will not operate statistically. For this reason it requires a turbojet or rocket assist to accelerate it to operating speed.

At supersonic speeds the ramjet engine is capable of producing very high thrust with high efficiency. This characteristic makes it quite useful on high speed aircraft and missiles, where its great power and low weight make flight possible in regions where it would be impossible with any other power plant except the rocket. Ramjets have also been used at subsonic speeds where their low cost and light weight could be used to advantage.

Principle of Operation: The ramjet consists of a diffuser, fuel injector, flame holder, combustion chamber and exit nozzle (Ref figure 9). The air taken in by the diffuser is compressed in two stages.

The external compression takes place takes place because the bulk of the approaching engine forces the air to change its course. Further compression is accomplished in the diverging section of the ramjet diffuser. Fuel is injected into and mixed with air in the diffuser. The flame holder provides a low velocity region favourable to flame propagation, and the fuel-air mixture recirculates within this sheltered area and ignites the fresh charge as it passes the edge of the flame holder. The burning gases then pass through the combustion chamber, increasing in temperature and therefore in volume. Because the volume of air increases, it must speed up to get out of the way off the fresh charge following behind it, and a further increase in velocity occurs as the air is squeezed out through the exit nozzle. The thrust produced by the engine is proportional to this increase in velocity.

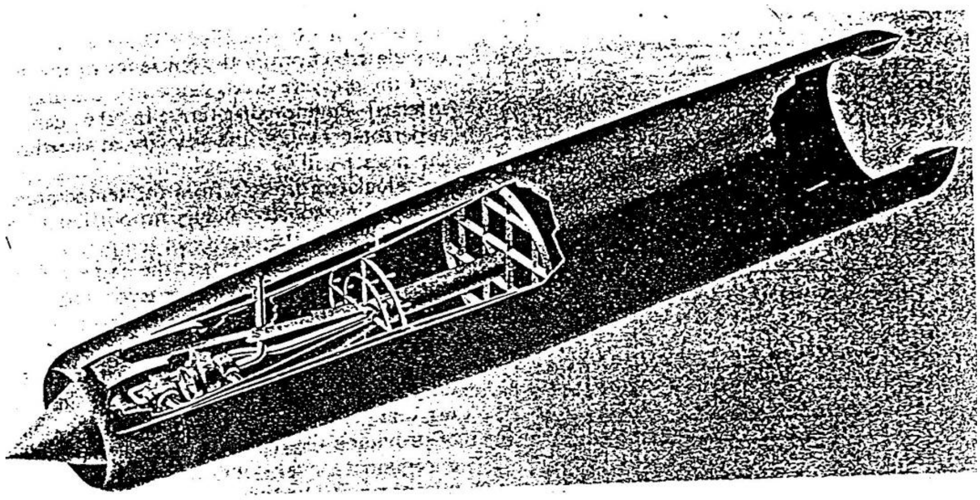


Figure 9 a Cutaway view of a supersonic ramjet.

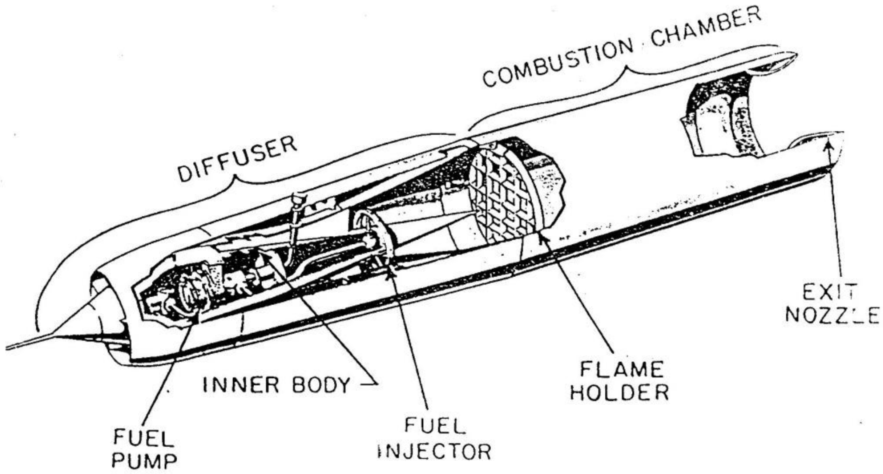


Figure 9 b Supersonic ramjet, showing components.

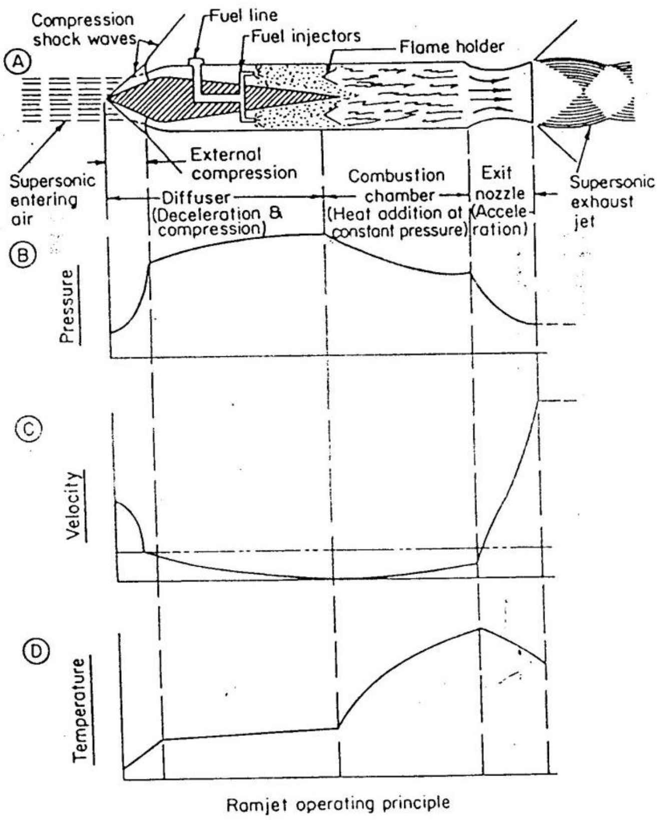
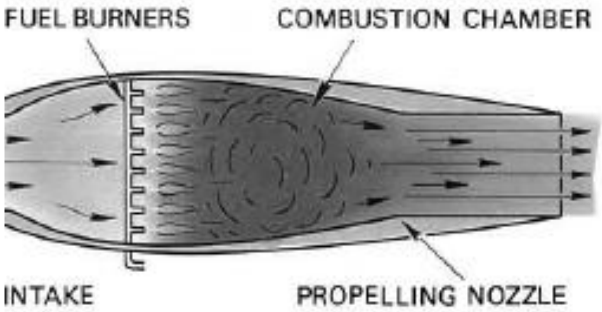


Figure — 9c- Operating conditions within a supersonic ramjet.



1-6 A ram Jet engine.

Advantages

1. Ramjet is very simple and does not have any moving part. It is very cheap and requires almost no maintenance.

2. Since turbine is not used the maximum temperature which can be allowed in ramjet is very high, about 2000^0 C as compared to about 1000^0 C in turbojets. This allows a greater thrust to be obtained by burning fuel at A/F ratio of about 15.1 which gives higher temperatures.

3. The SFC is better than turbojet engines at high speed and high altitudes.

4. There seems to be no upper limit to the flight speed of the ramjet.

Disadvantages

1. Since the compression of air is obtained by virtue of its speed relative to the engine, the take-off thrust is zero and it is not possible to start a ramjet without an external launching device.

2. The engine heavily relies on the diffuser and it is very difficult to design a diffuser which will give good pressure recovery over a wide range of speeds.

3. Due to high air speed, the combustion chamber requires flame holder to stabilise the combustion.

4. At very high temperature of about 2000^0 C dissociation of products of combustion occurs which will reduce the efficiency of the plant if not recovered in nozzle during expansion.

Application: Due to its high thrust at high operational speed, it is widely used in high speed aircrafts and missiles. Subsonic ramjets are used in target weapons, in conjunction with turbojets or rockets for getting the starting torque.

PULSE JET ENGINE

The pulse jet engine is an intermittent, compressor less aerodynamic power plant, with few or none of the mechanical features of conventional aviation power plants. In its simplest form, the operation of the pulse jet depends only on the properties of atmospheric air, a fuel, a shaped tube and some type of flow-check valve, and not on the interposition of pistons, impellers, blades or other mechanical part whose geometry and motion are controllable. The pulse jet differs from other types of air breathing engines, in that the air flow through it is intermittent. It can produce static thrust.

Operations: (Ref. Figure 10) during starting compressed air is forced into the inlet which opens the spring loaded flapper valves. In practice this may done by blowing compressed air though the valve box or by the motion of the engine through the air. The air that enters the engine passes by the fuel injector and is mixed with the fuel(Fig. A)

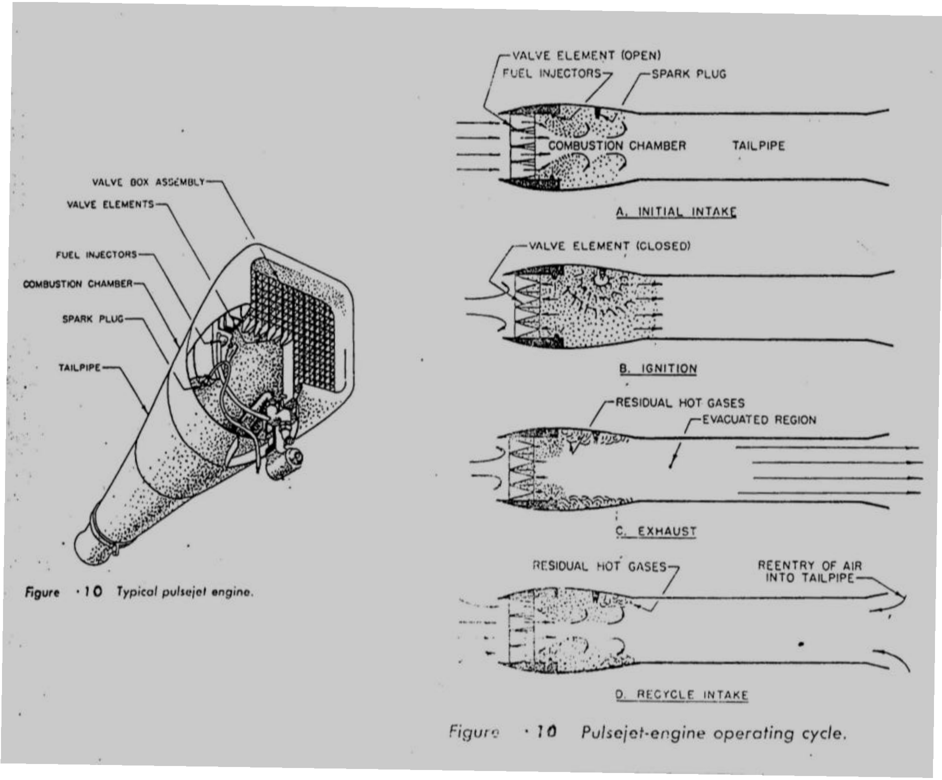


Figure ·10 Typical pulsejet engine.

Figure ·10 Pulsejet-engine operating cycle.

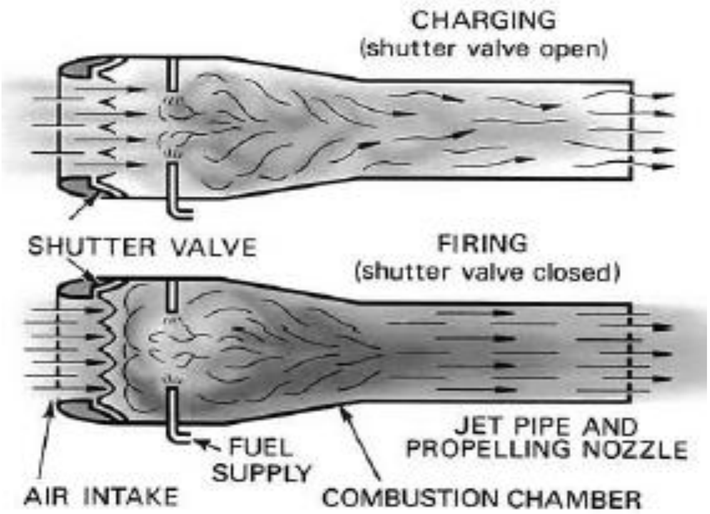


Fig. 1-7 A pulse jet engine.

When the fuel-air mixture reaches the proper proportion to burn, it is ignited by a spark plug. The burning takes place with explosive force, thus causing a very rapid rise in pressure, the increase in pressure forces the flapper valves shut and propels the charge of burned gases out of the tail pipe, as in B of the figure.

The momentum of the gases leaving the tailpipe causes the air to continue t flow out even after the pressure within the engine has reached atmospheric pressure. The pressure within the engine is therefore evacuated to below atmosphere, part C in figure.

After the pressure has reached its lowest point, atmospheric pressure (and the ram pressure if the engine is in flight) forces air into the engine through the flapper valves. At the same time, air will also be drawn in through tailpipe, since the pressure within the tailpipe is low and has nothing to prevent the entry of air, At this point, part D in figure, the engine is ready to begin another cycle. The fequency of cycles depends upon the duct shape and working temperature in V-1 rocket it was about 40 c/s which corresponds to about 2400 rpm of a two stroke reciprocating engine.

Once the engine operation has become established, the spark plug is no longer necessary. The reignition between each cycle is accomplished when the fresh charge of fuel and air is ignited by some residual flame which is left over from the preceding cycle. The air flow which reenters the tailpipe is important from both the engine operation and thrust stadpoints. Experiments have shown that the amount of air which flows into the tailpipe can be several times as much as that which flows into the inlet. This mass flow of air increases the thrust of the engine by providing additional mass for the explosion pressure to work on. It also increases the pressure within the engine at the beginning of each explosion cycle, resulting in a more efficient burning process. Reentry of air into the tailpipe is made more difficult as the airspeed surrounding the engine increases. The thrust of the engine, therefore, tends to decrease with speed. As the speed increases, the amount of reentering air flow decreases to the point where the internal pressure is eventually too low to support combustion and the engine will no longer operate.

Characteristics : The chief advantages of the pulse jet are its simplicity, light weight, low cost and good zero speed thrust characteristic. Its particular disadvantages are its 650-800 km/h. operating speed limit, rather limited altitude range and somewhat unpredictable valve life.

One interesting and sometimes objectionable, feature of the pulse jet engine is the sound it makes when in operation. The sound is a series of loud reports caused by the firing of the individual charges of fuel and air in the combustion chamber. The frequency of the reports depends upon the length of the engine form the inlet valves to the end of the tailpipe and upon the temperature of the gases within the engine. The resulting sound is a continuous, loud, and vibratory note that can usually be heard for several kilometers.

The pulse jet has low thermal efficiency. In early designs the efficiency obtained was about 2 to 3% with a total flight life of 30 to 60 minutes. The maximum operating speed is seriously limited by tow factors: (i) It is possible to design a good diffuser at high speeds. (ii) The fiepper valves, the only mechanical part in the pulse jet, also have certain natural frequency and if resonance with the cycle frequency occurs then the valve may remain open and no compression will take place. Also, as the speed increases it is difficult for air to flow back. This reduces total compression pressure as well as the mass flow of air which results in inefficient combustion and lower thrust. The reduction in thrust and efficiency is quite sharp as the speed increases.

Advantages :

1. This is very simple device only next to ramjet and is light in weight. It requires very small and occasional maintenance.

2. Unlike ramjet, it has static thrust because of the compressed air starting, thus it does not need a device for initial propulsion. The static thrust is even more than the cruise thrust.

3. It can run on an almost any type of liquid fuel without much effect on the performance. It can also operate on gaseous fuel with little modifications.

4. Pulse jet is relatively cheap.

Disadvantages :

1.The biggest disadvantage is very short life of flapper valve and high rates of fuel consumption. The SFC is as high as that of ramjet.

2. The speed of the pulse jet is limited within a very narrow range of about 650-800 km/h because of the limitations in the aerodynamic design of an efficient diffuser suitable for a wide range.

3. The high degree of vibrations due to intermittent nature of the cycle and the buzzing noise has made it suitable for pilotless crafts only.

4. It has lower propulsive efficiency that turbojet engine.

5. The operational range of the pulse jet is limited in altitude range.

Applications: German V-1 buzz bomb, American Helicopter company's Jet Jeep Helicopter, Auxiliary power plant for sail planes.

RAM ROCKET

Ram rocket (Ref. Figure 11) is an attempt to combine the initial thrust of a rocket with the lower fuel consumption of ramjet. Afterburner can also be used in a ramjet for thrust augmentation. But, the use of a rocket allows it to start itself from rest and accelerates it to its efficient operating speed range. The rocket is shut down at high speed. Due to high SFC of rocket, the ram rocket unit has very SFC at lower speeds.

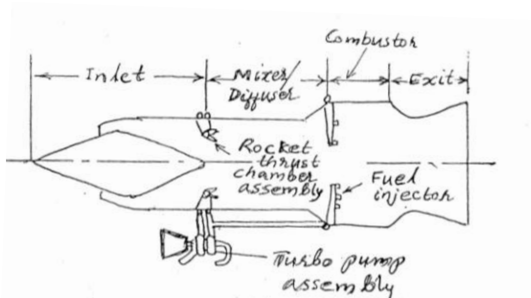
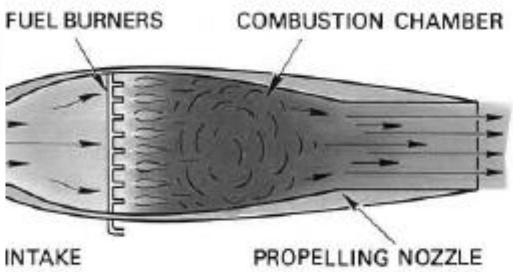


Figure 11



1-6 A ram Jet engine.

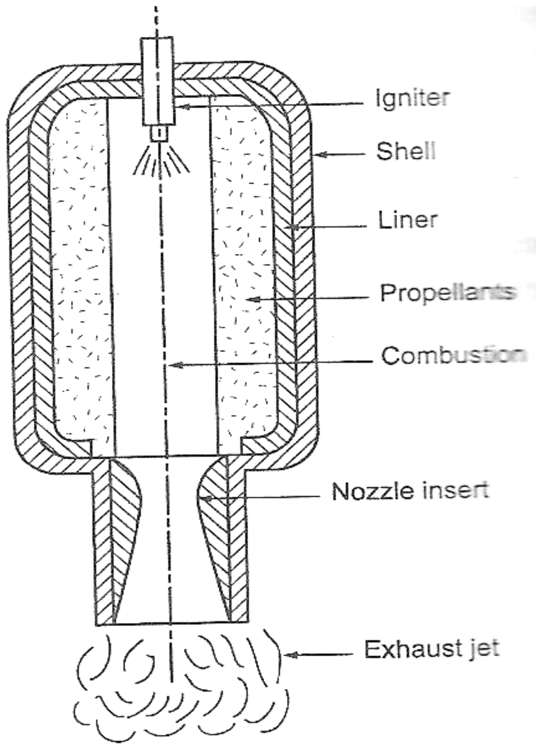
SOLD PROPELLANTS

Gas generated in a sold propellant is produced by reaction of reducing and oxidizing agent in the combustion chamber. The oxidizing agent is usually an inorganic slat or one or more organic nitro compounds. The reducing agent is usually a polymeric organic binder compound of carbon hydrogen and sometimes sulphur. The reducing agent imparts mechanical strength to the propellant.

SOLID PROPELLANT ROCKET ENGINE

Construction:

- The construction of solid propellant rocket engine is shown in figure:



- Solid propellant is the combination of solid fuel (plastic or resin material) and oxidizer

  (Nitrates & per-chlorates)

- Solid fuel & oxidizer is homogeneously mixed & packed inside the shell.
- A liner is provided between the shell and the propellant. The purpose of the liner is to protect the shell as high temperature will be generated during combustion process.

Working:

- The igniters is located at the top and ignites the spark. So combustion takes place.
- When the combustion take place in the combustion chamber , very high pressure and temperature gases are produced
- The highly heated products of combustion gases are then allowed to expand in the nozzle section.
- In the nozzle pressure energy of the gas is converted into kinetic energy. So the gases come out from the unit with very high velocities.
- Due to high velocities of gases coming out from the unit, a force or thrust is produced in opposite direction. This thrust propels the rocket.

Advantages:

- Simple in design and construction
- They do not require feed system. So they are free from the problems of moving parts such as pumps, valves etc
- Less vibration due to absence of moving parts.
- Less maintenance
- Suitable for short range applications.
- Problems arising from the sudden emptying of propellant tanks are absent.

Disadvantages:

- In case of emergency it is difficult to stop the engine in the midway.
- Decrease of speed is not possible
- Low specific impulse
- At the end of an operation, the burnt up debris cannot be reused. So it is uneconomical.
- Nozzle cooling is not possible.
- Nozzle erosion is unavoidable due to the presence of solid particles in the high temperature and high speed gases.

- Transportation and handling of these rockets before firing require due to the presence of propellant throughout.

SOLID PROPELLANTS

1. Heterogeneous or composite propellants
2. Homogeneous propellant

In heterogeneous solid propellants, plastic polymers and polyvinyl chlorides are used as fuels. Nitrates and per-chlorates are used as oxidizers. In homogeneous solid propellants, nitroglycerine and nitrocellouse are used. It combines the properties of fuel and oxidizers.

PROPERTIES OF SOLID PROPELLANTS

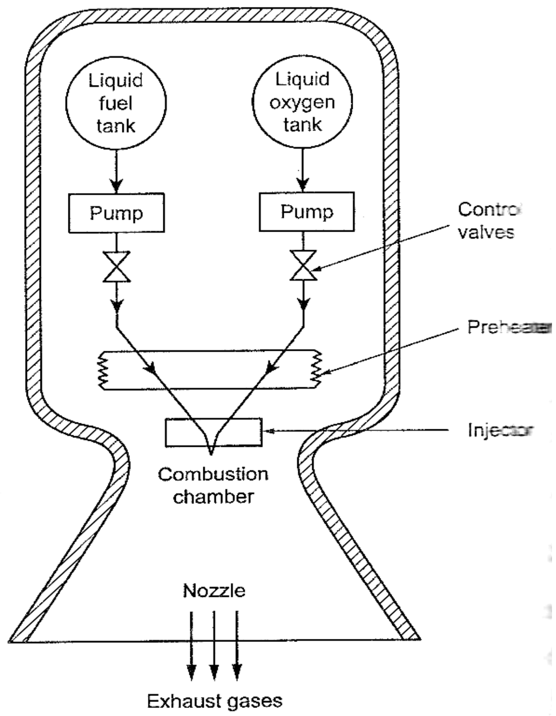
- Should release large amount of heat during combustion
- Physical & chemical properties should not change during processing.
- It should have high density
- Should not be poisonous and hazardous
- Should be cheap and easily available
- Should be noncorrosive and nonreactive with the components of the engine.
- Storage and handling should be easy.

LIQUID PROPELLANT ROCKET ENGINE

Construction

- The construction of liquid propellant rocket engine is shown in figure
- Liquid fuel(refined petrol, liquid hydrogen, hydrazine etc) and liquid oxygen are used in this engine
- Liquid fuel and liquid oxygen are stored separately in two different tanks
- Preheater is used to heat the fuel and oxidizer
- Nozzle is used to increase the velocity and decrease the pressure of the gases

Working



• Liquid fuel and liquid oxygen are pumped separately into a combustion chamber through control valves.

• Since the liquid fuel and liquid oxygen are stored at very low temperature , they are preheated in the preheated to a suitable temperature

• The preheater fuel-oxidizer mixture is injected into the combustion chamber through suitable injector and combustion takes place.

• When the combustion takes place in the combustion chamber , very high pressure and temperature gases are produced

• The highly heated products of combustion gases are then allowed to expand in nozzle section.

• In the nozzle, pressure energy of the gas is converted into kinetic energy. So the gases coming out from the unit with the very high velocity.

• Due to high velocity of gases coming out from the unit , a force or thrust is produced in the opposite direction. This thrust propels the rocket.

Advantages

- Liquid propellant engines can be reused after recovery. So it is economical .
- Combustion process is controllable.
- Speed regulation is possible
- High specific impulse
- More economical for long range operation
- Malfunctions and accidents can be rectified at any stage

Disadvantages

- It's construction is more complicated compared to solid propellant rock
- There are additional handling and safety problems if the propellants are poisonous and corrosive
- Manufacturing cost is high
- High vibration
- Size and weight of the engine is more compared to solid propellant rockets
- Any liquid propellants can exist in liquid state at very low temperature. So proper insulation is needed

LIQUID PROPELLANTS

a) Monopropellants

b) Bipropellants

a) Monopropellant

A liquid propellant which contains both fuel and oxidizer in a single chemical is known as monopropellant. It stable at ambient conditions and liberates thermo-chemical energy on heating. Monopropellants have been widely used in solid propellant rockets.

Examples : Nitro glycerin, nitro methane, Hydrogen peroxide, Hydrazine

b) Bipropellants

If the fuel and oxidizer are different from each other in its chemical nature then the propellant is called bipropellant. Bipropellants have been widely used in liquid propellant rocket system.

PROPERTIES OF LIQUID PROPELLANTS

- Propellant should have high calorific value
- Its density should be high
- It should have low values of vapor pressure and viscosity
- It should have higher specific heat and thermal conductivity
- Products of combustion should have low molecular weight to produce high jet velocity
- It should be non corrosive and non reactive with components of the engine
- It should not be poisonous and hazardous
- It should be cheap and easily available
- Energy released during combustion per unit mass of the propellant combination should be high
- It should be easily ignitable

Solid Propellant Rockets (SPR)

In solid propellant rockets the propellant to be burned is contained within the combustion chamber of case. The propellant charge is called the grain and it contains all the chemical elements for complete burning. Once ignited, it usually burns smoothly at nearly constant rate on the exposed surface of the charge. Because there are no feed systems or valves, such as there in liquid units, solid propellant rockets are usually relatively simple in construction.(Ref. Fig. 12)

Advantages of SPR over LPR

1. Simpler in construction and design - Fuel and oxidiser are in one mass inside the combustion chamber and there are no moving parts like pumps as in the case of LPR.

2. SPR can be handled with greater ease in the field - there are no special storage problems, no deterioration during storage and less danger due to explosion.

3. Lower initial cost - even though solid propellants are costlier the overall cost is less.

4. Excellent reliability - chances of malfunctioning are less.

5. Low servicing problems

6. For low total impulse applications it is lighter.

7. Much easier in achieving multistaging or clustering with SPR.

Disadvantages :

1. Cooling of the combustion chamber and the throat of the nozzle is a big problem.

2. On off control is not possible - difficult to extinguish and reignite, since the control of propellant is not in our hand.

3. Specific impulse is lower than LPR engines.

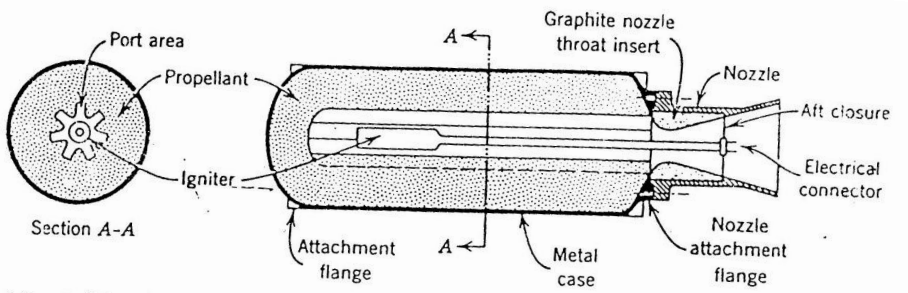
4. For long duration or high thrust it becomes heavier.

5. Thrust cannot be varied readily.

6. Do not have rapid thrust vector control. The direction of flow from the nozzle is not easy to deflect.

Liquid Propellant Rockets (LPR)

Liquid propellant rocket use liquid propellant that are fed under pressure from tanks into a thrust chamber. In the thrust chamber the propellant react to form hot gases, which in turn are accelerated and ejected at a high velocity through a supersonic nozzle, thereby imparting momentum to the system. A liquid rocket unit usually permits repetitive operation and can be started and shut-off at will. If the thrust chamber is provided with adequate cooling capacity, it is possible t run liquid rockets for periods extending one hour, dependent only on the propellant supply. A liquid propellant rocket system, is however, relatively complicated; it requires several precision valves and a complex feed mechanism which often includes propellant pumps, turbines or a propellant pressurising device, and a relatively intricate combustion or thrust chamber.



ig. 12 Solid rocket propellant motor. Here the propellant grain is bonded to the case. The grain as a seven-pointed star configuration.

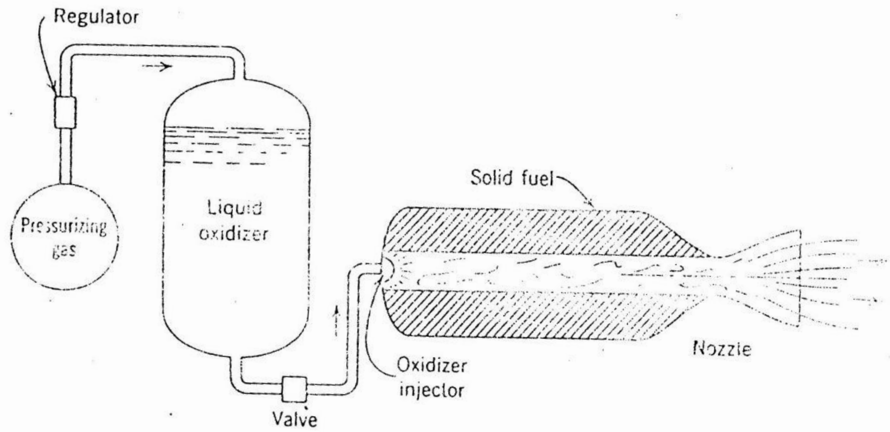


Fig. 15 Schematic diagram of typical hybrid rocket engine.

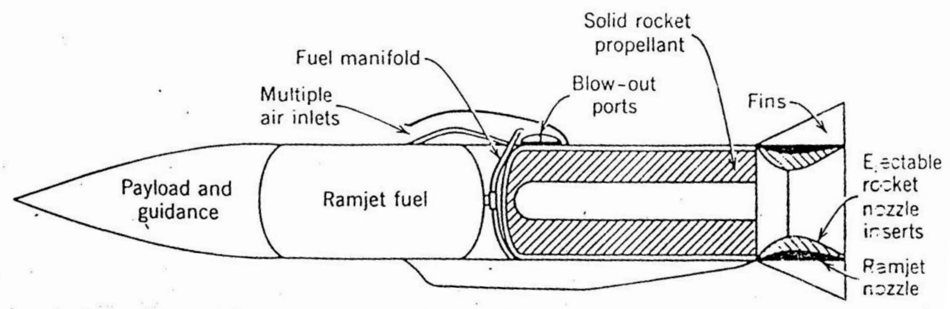


Fig. 16 Elements of an air-launched missile with integral rocket-ramjet propulsion.

Pressure Feed System

One of the simplest and most common means of pressurising the propellant is to force them out of their respective tanks by displacing them with high pressure gas. The gas is fed into the propellant tanks at a controlled pressure, thereby giving a controlled propellant discharge.

For low thrust and/or short duration, such as for space vehicles or anti-aircraft rockets, a feed system of this type is preferred. The rocket engines with pressurised feed system can be very reliable because of their simplicity.

A simple pressurised feed system is shown schematically in Figure 13. It consists essentially of a high pressure gas tank, a gas shut-off and starting valve, a pressure regulator, propellant tanks, propellant valves and feed lines. Additional components such as filling and draining provisions, check valves and filters are also often incorporated.

After all tanks are filled, the high-pressure air valve is remotely actuated and admits air through the pressure regulator at a constant pressure to the propellant tanks. The purpose of the check valves is to prevent mixing of the oxidiser with the fuel when the unit is not in the upright position. The propellant are fed to the thrust chamber by opening valves. When the propellant are completely consumed, the pressuring air serves also as a scavenging agent and cleans lines and valves of liquid propellant residue.

Turbo pump Feed System

The turbo pump rocket feed system pressurizes the propellant by means of pumps, which in turn are driven by turbines. The turbines derive their power from the expansion of hot gases. (Ref. Fig. 14)

Turbopump rocket systems are usually used on high thrust and long duration rocket units; they are lighter than other types for these applications. Their engine weight is essentially independent of thrust.

Advantages of LPR over SPR

1. Duration of operation can be controlled.

2. Size of the combustion chamber is small as whole of the fuel need not be stored in it. (The fuel and oxidiser are stored outside in separate tanks)

3. On and off control is possible.

4. Use of cooling allows the thrust chamber walls t maintain their strength, use of less expensive or non-critical materials and preheating of the fuel.

5. Control of LPR is easier than control of SPR.

Hybrid Rockets

Hybird rockets make use of various combinations of solid and liquid propellants. Most common is the liquid oxidiser-solid fuel concept shown in figure. The oxidiser can be either a storable of a cryogenic liquid depending on the specific impulse or other requirements of the application. RFNA (Red fuming nitric acid) and a fuel grain of polybutandiene-polymethyl methacrylate) are typical low cost hybrid propellants. Actual specific impulse valves are between 170 and 220s for storable oxidisers and plastic fuels (Ref. Fig. 15)

The main advantages of a hybrid rocket are : (a) low cost for applications where economy is essential and low performance is acceptable. (b) simplicity of stored grain fuel. (c) a liquid for nozzle cooling and thrust modulation. (d) start-up-restart capabilities. (e) good storability traits and (f) safety during storage or operation.

Integral rocket-ramjet

The principles of rocket and ramjet can be combined so that the two propulsion systems operate in sequence and in tandem and yet utilise a common combustion chamber volume as shown in Figure 16. The low-volume configuration, known as an integral rocket-ramjet, is particularly attractive in air launched missiles using ramjet propulsion. The transition from the rocket to the ramjet requires enlarging the exhaust nozzle (usually by ejecting rocket nozzle parts), opening the ramjet air inlet-combustion chamber interface, and following these two events with the normal ramjet starting sequence.

Application of Rockets

1. Airplane power plants:

(a) Primary power plants eg:- German Mel 163 fighter used in II World war

X- 1 Research engine (first to break the sonic barrier)

X - 15 supersonic research aircraft

(b) Auxiliary power plants for super performance or improving the performance (speed, rate of climb), assisted take-off.

2. Weapons

(a) Rocket projectiles (unguided missile) - not accurate, payloads explosive charges, smoke charge, other military payloads (or mail carriers to reach remote villages in mountain areas)

(b) Guided missiles:- similar to rocket projectiles but bigger in size and the trajectory is controlled - ground to ground, ground to air (aircraft), air to air, air to ground, ship to air and ship to ship missiles - payloads are atomic weapons.

3. Space vehicles

(a) military purposes - reconnaissance

commercial purposes - communication and weather studies

scientific purposes - lunar, space and planets

(b) Manned and unmanned space vehicles

(c) Near earth, longer, interplanetary, trains- solar systems

Rocket engines are use to (1) take-off from earth, ascend and achievement of orbit (2) altitude control, trajectory connections, reentry, attainment of lunar and planetary orbits and landing, separation of vehicle stages, maneuvers etc.

4. Sounding rockets - rockets which carry instruments to measure meteorological and scientific data at high altitudes, may be guided or unguided.

5. Other applications - for throwing life line to ships, signal rockets, antitank rockets, under water rockets etc.

6. Miscellaneous - ejection or crew escape capsules and stores personnel “propulsion belts”,

Propulsion for target drones, signal rockets, decoy rockets, spin rockets, vernier rockets, under water rockets for torpedoes and missiles, the throwing of life lines to ships and “Fourth of July rockets”

THRUST

The force which propels the air craft forward at a given speed is called as thrust or propulsive force. This thrust mainly depends on the velocity of gases at the exit of the nozzle.

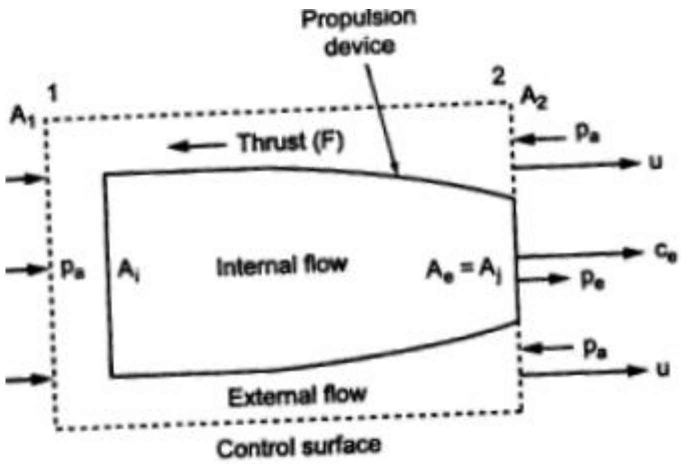
JET THRUST

The control surface of a turbojet engine between section 1 and 2 is shown in figure.

Air from atmospheric ($\dot{m}_a$) enters the Turbojet engine at a pressure P_a and velocity u.

The gases leaves the nozzle at a pressure of P_e and velocity of C_e..

The mass flow rate of gases at exit of the nozzle is $\dot{m}_a + \dot{m}_f$.



We know that

Net thrust of the engine (F) = $\{Momentum\ thrust\ (F_{mom})\} + \{Pressure\ thrust\ (F_{pre})\}$

$$Momentum\ thrust\ (F_{mom}) = (\dot{m}_a + \dot{m}_f)c_e - \dot{m}_a \times u$$

$$Pressure\ (F_{pr}) = (P_e - P_a) \times A_\varepsilon$$

$$Net\ thrust\ (F) = (\dot{m}_a + \dot{m}_f)c_e - \dot{m}_a \times u + (P_e - P_a) \times A_\varepsilon$$

$For\ complete\ expansion,$

$$P_a = P_e.$$

$$net\ thrust\ F = (\dot{m}_a + \dot{m}_f)c_e - \dot{m}_a \times u$$

Where

$m^{\bullet}_{a}$ – *Mass of air* (kg/s)

$m^{\bullet} f$ – *Mass of fuel* (kg/s)

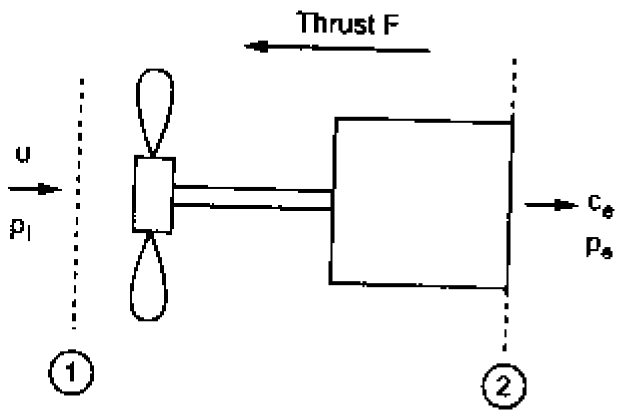
c_e – *Exit velocity of gases*

u – *Flight speed* (m/s)

PROPELLER THRUST

The control surface of a turbo prop engine between section 1 and 2 is shown in the figure

We know that,



Net thrust considering mass of fuel $F = \left(m^{\bullet}_{a} + m^{\bullet}_{f}\right) c_e - m^{\bullet}_{a} \times u$

$m^{\bullet} c_{e} - m^{\bullet}_{a} \times u$

For complete expansion,

$c_e = c_j$

$$\boxed{F = m^{\bullet} c_j - m^{\bullet}_{a} \times u}$$

Since $m^{\bullet}_{f}$ *is very small compared to* $m^{\bullet}_{a}$, *it is neglected.*

Net thrust without considering mass of fuel $F = m^{\bullet} c_j - m^{\bullet} u$

$$\boxed{\begin{array}{l} F = m^{\bullet} \ (c_j - u) \\ \qquad or \\ F = m^{\bullet}_{a} \ (c_j - u) \end{array}}$$

EFFECTIVE SPEED RATIO (σ)

The ratio of flight speed to jet velocity is known as effective speed ratio (σ)

$$\sigma = \frac{Flight\ speed}{Jet\ velocity\ (or)\ Velocity\ of\ exit\ gases}$$

$$\boxed{\sigma = \frac{u}{c_j}}$$

We know that, $\quad Thrust\ \ F = m^{\bullet}_{a}\left[c_j - u\right]$

$$= m^{\bullet}_{a} \times c_j \left[1 - \frac{u}{c_j}\right]$$

$$\boxed{F = m^{\bullet}_{a} \times c_j \left[1 - \sigma\right]}$$

SPECIFIC THRUST (F_{sp})

The thrust developed per unit mass flow rate is known as specific thrust

$$\boxed{F_{sp} = \frac{F}{m^{\bullet}}}$$

THRUST SPECIFIC FUEL CONSUMPTION (TSFC)

The fuel consumption rate per unit thrust is known as Thrust Specific Fuel Consumption

$$\boxed{TSFC = \frac{m^{\bullet}_{f}}{F}}$$

SPECIFIC IMPULSE (I_{sp})

The thrust developed per unit weight flow rate is known as specific impulse

$$I_{sp} = \frac{F}{w}$$

$$= \frac{m^{\bullet}(c_j - u)}{m^{\bullet} \times g}$$

$$\frac{c_j - u}{g} = \frac{u}{g}\left[\frac{c_j}{u} - 1\right]$$

$$I_{sp} = \frac{u}{g}\left[\frac{1}{\sigma} - 1\right]$$

Where, $\sigma - \text{effective speed ratio} = \frac{u}{c_j}$

PROPULSIVE EFFICIENCY

It is defined as the ratio of Propulsive power (or) thrust power to the power output of the engine.

$$\eta_P = \frac{\text{Propulsive power (or) Thrust power}}{\text{Power output of the engine}}$$

We know that

$$\text{Thrust power} = \text{Thrust}(F) \times \text{Flight speed}(u)$$

$$\boxed{\text{Thrust power} = m^{\bullet}(c_j - u) \times u}$$

At the outlet of the engine, the power is available in the form of kinetic energy. So the power output of the engine is $\frac{1}{2}m^{\bullet}\left[c^2{}_j - u^2\right]$

$$\boxed{\text{Power output} = \frac{1}{2}m^{\bullet}\left[c^2{}_j - u^2\right]}$$

$$\eta_P = \frac{m^{\bullet}\left[c_j - u\right] \times u}{\frac{1}{2}m^{\bullet}\left[c^2{}_j - u^2\right]}$$

$$\eta_P = \frac{\left[c_j - u\right] \times u}{\frac{1}{2}\left[c^2{}_j - u^2\right]} = \frac{2u\left[c_j - u\right]}{c^2{}_j - u^2}$$

$$= \frac{2u\left[c_j - u\right]}{(c_j + u)\,(c_j - u)}$$

$$\boxed{\eta_P = \frac{2u}{c_j + u}}$$

Divide the numerator and denominator by c_j.

$$\eta_P = \frac{\frac{2u}{c_j}}{\frac{c_j + u}{c_j}} = \frac{\frac{2u}{c_j}}{1 + \frac{u}{c_j}}$$

$$\boxed{\eta_P \ \frac{2\sigma}{1+\sigma}}\ \textit{Where}$$

$$\sigma - \textit{Effective speed ratio} = \frac{u}{c_j}$$

THERMAL EFFICIENCY

It is defined as the ratio of power output of the engine to the power input to the engine

$$\eta_t = \frac{\textit{Power output of the engine}}{\textit{Power input to the engine through fuel}}$$

Power is given as the input by burning the fuel

So, $\quad \textit{Power input} = m^{\bullet}{}_f \times C \cdot V$

We know that

$$\textit{Power output} = \frac{1}{2} m^{\bullet} \left[c^2{}_j - u^2\right]$$

$$\eta_t = \frac{\frac{1}{2} m^{\bullet} \left[c^2{}_j - u^2\right]}{m^{\bullet}{}_f \times C \cdot V}$$

Where

$m^{\bullet}$ – Mass of air fuel mixture

c_j – Velocity of jet

u – Flight velocity

$m^{\bullet}{}_f$ – Mass of fuel

$C \cdot V$ – Calorific value of fuel

If efficiency of combustion is considered,

$$\eta_t = \frac{\frac{1}{2} m^{\bullet} \left[c^2{}_j - u^2 \right]}{\eta_B \times m^{\bullet}{}_f \times C \cdot V}$$

OVERALL EFFICIENCY

It is defined as the ratio of propulsive power to the power input to the engine

$$\eta_0 = \frac{\text{Propulsive Power (or) Thrust Power}}{\text{Power input to the engine}}$$

We know that,

$$\text{Thrust power} = m^{\bullet} \left[c_j - u \right] \times u$$

$$\text{Power input} = m^{\bullet}{}_f \times C \cdot V$$

$$\eta_0 = \frac{m^{\bullet} \left[c_j - u \right] \times u}{m^{\bullet}{}_f \times C \cdot V}$$

$$= \frac{m^{\bullet} \left[c_j - u \right] \times u}{\frac{1}{2} m^{\bullet} \left[c^2{}_j - u^2 \right]} \times \frac{\frac{1}{2} m^{\bullet} \left[c^2{}_j - u^2 \right]}{m^{\bullet}{}_f \times C \cdot V}$$

$$\eta_P \times \eta_\tau$$

$$\boxed{\eta_0 = \eta_P \times \eta_\tau}$$

Question 1: An aircraft flies at a speed of 520 kmph at an altitude of 8000 m. The diameter of the propeller of an aircraft is 2.4 m and flight to jet speed ratio is 0.74. Find the following:

(i) The rate of air flow through the propeller

(ii) Thrust produced

(iii) Specific thrust

(iv) Specific impulse

(v) Thrust power

Given:

Air craft speed (or) Flight speed = 520 kmph

$$= \frac{520 \times 10^3}{3600\ s}$$

$= 144.44\ m/s$

$Altitude\ \ z = 8000\ \ m$

$Diameter\ of\ the\ propeller\ d\ = 2.4\ m$

$Flight\ to\ jet\ speed\ ratio\ \sigma\ = \frac{u}{c_j}\ = 0.74$

$Where\ \ c_j\ -\ jet\ speed\ (or)\ Speed\ of\ exit\ gases\ from\ the\ engine$

$Solution\ :\ Area\ of\ the\ propeller\ disc\ A\ = \frac{\pi}{4}\, d^2$

$$= \frac{\pi}{4}\ \ (2.4)^2$$

$$= \boxed{A\ = 4.52\ m^2}$$

$From\ gas\ tubes\ \ at\ Z = 8000m$

$\rho\ = 0.525\ kg/m^3$

$Effective\ speed\ ratio\ \ ,\sigma\ = \frac{u}{c_j}$

$$0.74 = \frac{144.44}{c_j}$$

$$\boxed{Velocity\ of\ jet, c_j = 195.19\ m/s}$$

Velocity of air flow at the propeller

$$c = \frac{1}{2}\left[u + c_j\right]$$

$$= \frac{1}{2}[144.44 + 195.19]$$

$$\boxed{c = 169.81\ m/s}$$

Mass flow rate of air – fuel mixture

$$m^{\bullet} = \rho\ A\ c$$

$$= 0.525 \times 4.52 \times 169.81$$

$$\boxed{m^{\bullet} = 402.96\ kg/s}$$

We know that

$$m^{\bullet} = m^{\bullet}{}_a + m^{\bullet}{}_f$$

since mass flow rate of fuel $(m^{\bullet}{}_f)$ is not given, let us take

$$m^{\bullet} = m^{\bullet}{}_a$$

$$\boxed{Mass\ flow\ rate\ of\ air\ m^{\bullet}{}_a = 402.96\ kg/s}$$

$$Thrust\ produced, F = m^{\bullet}{}_a\left[c_j - u\right]$$

$$= 402.96[195.19 - 144.44]$$

$$\boxed{F = 20.45 \times 10^3\ N}$$

$$Specific\ thrust\ F_{sp} = \frac{F}{m^{\bullet}}$$

$$= \frac{F}{m^{\bullet}{}_{a}}$$

$$= \frac{20.45 \times 10^{3}}{402.96}$$

$$\boxed{F_{sp} = 50.75 \ N \ / \ (kg/s)}$$

$$Specific \ impulse \ (I_{sp}) = \frac{F}{W}$$

$$= \frac{F}{m^{\bullet} g}$$

$$= \frac{F}{m^{\bullet}{}_{a} \times g}$$

$$= \frac{20.45 \times 10^{3}}{402.96 \times 9.81}$$

$$\boxed{I_{sp} = 5.17 \ s}$$

$$Thrust \ \ power \ , P = Thrust(F) \times \ Flight \ speed \, (u)$$

$$= 20.45 \times 10^{3} \times 144.44$$

$$P = 2.95 \times 10^{6} \ W$$

$$\text{Re}sult: \ (i) \ m^{\bullet}{}_{a} \ = 402.96 \ kg/ \ s$$

$$(ii) \ F \ = 20.45 \times 10^{3} \ N$$

$$(iii) \boxed{F_{sp} = 50.75 \ N \ / \ (kg/s)}$$

$$(iv) \boxed{I_{sp} = 5.17 \ s}$$

$$(v) \boxed{p = 2.95 \times 10^{6} W}$$

A rocket moves with a velocity of 10,000 km/hr with an effective exhaust velocity of 1400 m/sec, the propellant flow rate is 5 kg/sec and the propellant mixture has a heating value of 6500 kJ/kg. Find

1. Propulsion efficiency
2. Engine output power
3. Thermal efficiency
4. Overall efficiency

Sol)

Rocket speed, $u = 10{,}000 \text{km/hr}$

$$= \frac{10{,}000 \times 1000}{3600} \text{ m/sec}$$

$$= 2777.7 \text{ m/sec}$$

Jet velocity, $C_j = 1400$ m/sec

Calorific value (C.V.) = 6600 kJ/kg

Propellant flow rate m_p = 5 kg/sec

Speed ratio, $\sigma = \frac{u}{c_j} = \frac{2777.7}{1400} = 1.984$

Propulsion efficiency, $\eta_p = \frac{2 \times \sigma}{1+\sigma^2}$

$$= \frac{2 \times 1.984}{1+(1.984)^2} = .804$$

$$\eta_p = 80.4\ \%$$

Thrust, $F = m_p \times c_j$

$$= 5 \times 1400$$

$$= 7000 \text{ N}$$

Propulsive power, $P = F \times u$

$$= 7000 \times 2777.7$$

$$P = 19.44 \times 10^6 \text{ W}$$

Propulsive efficiency $= \frac{\text{Propulsive power}}{\text{Power output of engine}}$

$$.804 = \frac{19.44 \times 10^6}{\text{Power output of engine}}$$

Engine output $= 24.18 \times 10^6$ W

Thermal efficiency, $\eta_t = \frac{\text{Power output of engine}}{\text{Power input to engine}}$

$$= \frac{24.18 \times 10^6}{m_p \times C.V}$$

$$= \frac{24.18 \times 10^6}{5 \times 6600 \times 10^3} = .733$$

$\eta_t = 73.3\ \%$

Overall efficiency, $\eta_o = \eta_t \times \eta_p$

$$= .733 \times .804$$

$$= .589$$

$\eta_o = 58.9\ \%$