CALCULATING DERIVATIVES OF INVERSE FUNCTIONS

$f(x) = x^{1/n}$ on $[0, \infty)$

CONSIDER $0 < c < \infty$; LET $g(x) = x^n$ on $[0, \infty)$

OBSERVE THAT $f(x) = g^{-1}(x)$

$g'(c) = nc^{n-1} \neq 0$, SO BY THE THEOREM

ON DERIVATIVES OF INVERSE FUNCTIONS

$$(g^{-1})'(g(c)) = \frac{1}{g'(c)} = \frac{1}{nc^{n-1}}$$

WRITE $g(c) = d$, i.e. $c^n = d \Rightarrow c = d^{1/n}$; SO

$$f'(d) = \frac{1}{nc^{n-1}} = \frac{1}{n} d^{1/n - 1}$$

$f(x) = \sin^{-1} x \qquad g(x) = \sin x$ on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$f : [-1,1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, AND $f = g^{-1}$,

SO BY THE DERIVATIVE-OF-INVERSE THEOREM,

$$f'(x) = \frac{1}{g'(\theta)}, \text{ WHERE } g(\theta) = x$$

$$= \frac{1}{\cos\theta},$$

$\sin^2\theta = x^2$

$\Rightarrow \cos^2\theta = 1 - x^2$

$\Rightarrow \cos\theta = \sqrt{1-x^2}$

since $\cos\theta \geq 0$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$= \frac{1}{\sqrt{1-x^2}}$$

MONOTONICITY, LOCAL MAX/MIN:

WE SAY THAT f HAS A LOCAL MAXIMUM AT c IF THERE EXISTS $\delta > 0$ S.T. FOR ALL $x \in (c-\delta, c+\delta)$, $f(x) \leq f(c)$

A SIMILAR DEFINITION FOR LOCAL MINIMUM